Services du Premier Ministre
Programmation de la Politique scientifique
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1040 BRUXELLES
BELGIQUE

ACTIONS DE RECHERCHE CONCERTEES

ACTION INTERUNIVERSITAIRE

OCEANOLOGIE

Rapports des journées d'études
(1979)

Diensten van de Eerste Minister
Programmatie van het Wetenschapsbeleid
Wetenschapsstraat 8
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Shear effect dispersion in a shallow tidal sea

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Introduction

The hydrodynamics of shallow continental seas like the North Sea is dominated by long waves, tides and storm surges, with current velocities of the order of 1 m/s . The currents generate strong three-dimensional turbulence and vertical mixing, resulting, in general, in a fairly homogeneous distribution of temperature, salinity and concentrations of marine constituents over the water column.

Vertical gradients of concentrations may exist in localized area where vertical mixing is partly (and temporarily) inhibited by stratification or during short periods of time - a few hours following an off-shore dumping, for instance - before vertical mixing is completed. However such cases are very limited in space and time and, in most problems, it is sufficient to study, in a first approach, the horizontal distribution of depth-averaged concentrations.

If c denotes the concentration of a given constituant, the three-dimensional "dispersion" equation, describing the evolution of c in space and time, can be written (e.g. Nihoul, 1975).

$$\frac{\partial c}{\partial t} + \underset{\sim}{\nabla} . (c\underset{\sim}{v}) = Q + I - \underset{\sim}{\nabla} . (\underset{\sim}{\sigma} c) + D \qquad (1)$$

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In eq. (1),

i) $\underset{\sim}{\nabla}.(c\underset{\sim}{v})$ represents advection and can be separated in two parts corresponding respectively to the horizontal transport $\underset{\sim}{\nabla}.(c\underset{\sim}{u})$ and to the vertical transport $\frac{\partial}{\partial x_3}(cv_3)$; $\underset{\sim}{u} = v_1 \underset{\sim}{e}_1 + v_2 \underset{\sim}{e}_2$ denoting the horizontal current velocity.

ii) Q represents the rate of production (or destruction) of the constituent by volume sources (or sinks).
[*In most practical applications, inputs and outputs are located at the boundaries – in which case, they appear in the boundary conditions and no in* Q –, *or are localized quasi-instantaneous releases which may be conveniently taken into account in the initial conditions. In the following, one shall assume that this is the case and one shall set* $Q = 0$].

iii) I represents the rate of production (or destruction) of the constituent by (chemical, ecological, ...) interactions inside the marine system and I is, in general, a function of coupled variables c', c'', ...
[*A marine constituent is said to be passive when its evolution is not affected by such interactions. In the following, to simplify the formulation, one shall restrict attention to passive constituent and set* $I = 0$. *The generalization of the theory to a system of interacting constituents presents no fundamental difficulty (Nihoul and Adam, 1977)*].

iv) $-\underset{\sim}{\nabla}.(\underset{\sim}{\sigma}c)$ represents "migration". (sedimentation, horizontal migration of fish, ..., e.g. Nihoul, 1975).
[*Migration, at least in the most frequent case of sedimentation, can easily be taken into account (e.g. Nihoul and Adam, 1977). However, to avoid overloading the analysis, one shall assume, in the following, that the constituent is simply transported by the fluid and that the migration velocity* σ *is zero*].

v) D represents turbulent diffusion and can be separated into a vertical turbulent diffusion and a horizontal turbulent diffusion.
[*The horizontal turbulent diffusion is negligible as compared to the horizontal advection. The horizontal dispersion which is observed in the sea is mainly the result of the horizontal transport of the*

constituent by irregular and variable currents constituting a form of "pseudo horizontal turbulence" extending to much larger scales than the "proper" three-dimensional turbulence (e.g. Nihoul, 1975). In that case, D *can be simply written*

$$D = \frac{\partial}{\partial x_3} \left(\mu \frac{\partial c}{\partial x_3}\right) \tag{2}$$

where μ *is the vertical turbulent diffusivity*].

In the scope of the hypotheses made above, eq.(1) can be written, in the simpler form

$$\frac{\partial c}{\partial t} + \underset{\sim}{\nabla} . (c\underset{\sim}{u}) + \frac{\partial}{\partial x_3} (c v_3) = \frac{\partial}{\partial x_3} \left(\mu \frac{\partial c}{\partial x_3}\right) \tag{3}$$

The velocity field $\underset{\sim}{v} = \underset{\sim}{u} + v_3 \underset{\sim}{e}_3$ is given by the Boussinesq equations and in particular, one has

$$\underset{\sim}{\nabla} . \underset{\sim}{u} + \frac{\partial v_3}{\partial x_3} = 0 \tag{4}$$

Depth-averaged dispersion equation

Let

$$\bar{c} = H^{-1} \int_{-h}^{\zeta} c \; dx_3 \quad ; \quad \hat{c} = c - \bar{c} \qquad (5)\,;\,(6)$$

$$\bar{\underset{\sim}{u}} = H^{-1} \int_{-h}^{\zeta} \underset{\sim}{u} \; dx_3 \quad ; \quad \hat{\underset{\sim}{u}} = \underset{\sim}{u} - \bar{\underset{\sim}{u}} \qquad (7)\,;\,(8)$$

with

$$\int_{-h}^{\zeta} \hat{c} \; dx_3 = 0 \quad ; \quad \int_{-h}^{\zeta} \hat{\underset{\sim}{u}} \; dx_3 = 0 \qquad (9)\,;\,(10)$$

and

$$H = h + \zeta \tag{11}$$

where h is the depth and ζ the surface elevation.

One has

$$\frac{\partial \zeta}{\partial t} + \underset{\sim}{u}.\underset{\sim}{\nabla}\zeta = v_3 \qquad \text{at} \quad x_3 = \zeta \tag{12}$$

$$\frac{\partial h}{\partial t} + \underset{\sim}{u}.\underset{\sim}{\nabla}h = - v_3 \qquad \text{at} \quad x_3 = - h \tag{13}$$

Integrating eqs.(1) and (4) over depth, inverting the order of integration with respect to x_3 and of derivation with respect to t, x_1 or x_2 and using eqs.(12) and (13) to eliminate the corrections due to the variable limits of integration, one obtains (e.g. Nihoul; 1975)

$$\frac{\partial}{\partial t}\,(H\bar{c}) + \underset{\sim}{\nabla}.(H\,\bar{c}\,\bar{\underset{\sim}{u}}) + \underset{\sim}{\nabla}.\int_{-h}^{\zeta} \hat{c}\,\hat{\underset{\sim}{u}}\,dx_3 = 0 \tag{14}$$

$$\frac{\partial H}{\partial t} + \underset{\sim}{\nabla}.(H\,\bar{\underset{\sim}{u}}) = 0\ . \tag{15}$$

In the right-hand side of eq.(14), one should have the difference between the fluxes of the constituent at the free surface and at the bottom. The hypothesis is made here that there is no exchange between the water column and the atmosphere and between the water column and the bottom sediments.

In this case, combining eqs.(14) and (15), one gets

$$\frac{\partial \bar{c}}{\partial t} + \bar{\underset{\sim}{u}}.\underset{\sim}{\nabla}\bar{c} = \Sigma \tag{16}$$

where

$$\Sigma = H^{-1}\underset{\sim}{\nabla}.\int_{-h}^{\zeta} (-\,\hat{c}\hat{\underset{\sim}{u}})\;dx_3 \tag{17}$$

Σ contains the mean product of the deviations $\hat{c}$ and $\hat{\underset{\sim}{u}}$ around the mean values $\bar{c}$ and $\bar{\underset{\sim}{u}}$. The observations reveal that this term is responsible for a horizontal dispersion analogous to the turbulent dispersion but many times more efficient. This effect is called the "shear effect" because it is associated with the vertical gradient of the horizontal velocity $\underset{\sim}{u}$ (e.g. Bowden, 1965 ; Nihoul, 1975).

Parameterization of the shear effect

Substracting eq.(16) from eq.(1), one obtains

$$\frac{\partial \hat{c}}{\partial t} + \bar{\underset{\sim}{u}}.\underset{\sim}{\nabla}\hat{c} + \hat{\underset{\sim}{u}}.\underset{\sim}{\nabla}\hat{c} + \Sigma + v_3 \frac{\partial \hat{c}}{\partial x_3} + \hat{\underset{\sim}{u}}.\underset{\sim}{\nabla}\bar{c} = \frac{\partial}{\partial x_3}\left(\mu \frac{\partial \hat{c}}{\partial x_3}\right) \tag{18}$$

Because of the strong vertical mixing, one expects the deviation $\hat{c}$ to be much smaller than the mean value $\bar{c}$. This is not true for the velocity deviation $\hat{\underset{\sim}{u}}$ which may be comparable to $\bar{\underset{\sim}{u}}$; the velocity increasing from zero at the bottom to its maximum value at the surface. One may thus assume that the first four terms in the left-hand side of eq.(18) are negligible as compared to the sixth one $\hat{\underset{\sim}{u}}.\underset{\sim}{\nabla}\bar{c}$. The fifth term, representing vertical advection, is undoubtedly even smaller than the four neglected terms and eq.(18) reduces to

$$\hat{\underset{\sim}{u}}.\underset{\sim}{\nabla}\bar{c} = \frac{\partial}{\partial x_3}\left(\mu \frac{\partial \hat{c}}{\partial x_3}\right) \tag{19}$$

The physical meaning of this equation is clear : weak vertical inhomogeneities are constantly created by inhomogeneous convective transport and they adapt to that transport in such a way that the effects of advection and vertical turbulent diffusion are in equilibrium for them.

Integrating eq.(19) with the condition that the flux is zero at the free surface, one obtains

$$H\hat{\underset{\sim}{r}}.\underset{\sim}{\nabla}\bar{c} = \mu \frac{\partial \hat{c}}{\partial x_3} \tag{20}$$

where

$$\hat{\underset{\sim}{r}} = H^{-1} \int_{\zeta}^{x_3} \hat{\underset{\sim}{u}}\, dx_3 \tag{21}$$

Integrating by parts and taking into account that $\hat{\underset{\sim}{r}} = 0$ at $x_3 = \zeta$ and $x_3 = -h$ (cfr eq.10), one gets

$$\Sigma = H^{-1}\, \underset{\sim}{\nabla}.(H\, \underset{\approx}{R}\, .\, \underset{\sim}{\nabla}\, \bar{c}) \tag{22}$$

where $\underset{\approx}{R}$ is the shear effect diffusivity tensor, i.e. :

$$\underset{\approx}{R} = H \int_{-h}^{\zeta} \frac{\hat{\underset{\sim}{r}}\,\hat{\underset{\sim}{r}}}{\mu} \, dx_3 \tag{23}$$

To determine $\underset{\approx}{R}$, one must know the turbulent eddy diffusivity μ and the function $\hat{\underset{\sim}{r}}$, i.e. the velocity deviation $\hat{\underset{\sim}{u}}$.

Vertical profile of the horizontal velocity

The evolution equation for the horizontal velocity vector $\underset{\sim}{u}$ can be written, after eliminating the pressure (e.g. Nihoul, 1975)

$$\frac{\partial \underset{\sim}{u}}{\partial t} + \underset{\sim}{\nabla} . (\underset{\sim}{u}\,\underset{\sim}{u}) + f\underset{\sim}{e}_3 \wedge \underset{\sim}{u} + \frac{\partial}{\partial x_3} (v_3 \underset{\sim}{u}) = - \underset{\sim}{\nabla}\left(\frac{p_a}{\rho} + g\zeta\right) + \frac{\partial}{\partial x_3}\left(\nu \frac{\partial \underset{\sim}{u}}{\partial x_3}\right) \tag{24}$$

where f is equal to twice the vertical component of the earth's rotation vector, p_a is the atmospheric pressure, g the acceleration of gravity and ν the vertical turbulent viscosity.

In eq.(24), one has neglected the effect of the horizontal component of the earth's rotation vector (multiplied by $v_3 \ll u$) and the horizontal turbulent diffusion (because horizontal length scales are always much larger than the depth).

The observations indicate that, in shallow tidal seas, the turbulent viscosity ν can be written as the product of a function of t, x_1 and x_2 and a function of the reduced variable $\xi = H^{-1} (x_3 + h)$ (e.g. Bowden, 1965).

Let

$$\nu = H^2 \sigma(t,x_1,x_2) \; \lambda(\xi) \tag{25}$$

where σ and λ are appropriate functions.

The asymptotic form of ν for small values of ξ is well-known from boundary layer theory :

$$\nu = k \, u_* \, (x_3 + h) = k \, u_* \, H \, \xi \tag{26}$$

where k is the Von Karman constant and u_* the friction velocity given by

$$u_*^2 = \| \tau_b \| \qquad ; \qquad \tau_b = \left[\nu \frac{\partial u}{\partial x_3} \right]_{x_3 = -h} \tag{27);(28}$$

Hence

$$\sigma H = k u_* \tag{29}$$

and

$$\lambda(\xi) \sim \xi \qquad \text{for} \qquad \xi \ll 1 \, . \tag{30}$$

In a well-mixed shallow sea, where the Richardson number is small and the turbulence fully developed, it is reasonable (e.g. Nihoul, 1975) to take

$$\mu \sim \nu \, . \tag{31}$$

This hypothesis will be reexamined later.

It is convenient to change variables to (t, x_1, x_2, ξ) in eq.(24). In the final result (Nihoul, 1977), the non-linear terms combine with additional contributions from the time derivative to give three terms, related respectively to the gradients of velocity, depth and surface elevation. These terms are found negligible almost everywhere in the North Sea (Nihoul and Runfola, 1979). Thus although depth-integrated two-dimensional hydrodynamic models of the North Sea may not discard the non-linear terms[1], if one excludes localized singular regions like the vicinity of tidal emphydromic points, the "local" vertical distribution of velocity may be described, with a very good approximation, by a linear model.

Then, the governing equation for the velocity deviation $\hat{u}$ can be written

$$\frac{\partial \hat{u}}{\partial t} + f \, e_3 \wedge u = \sigma \left\{ \frac{\partial}{\partial \xi} \left[\lambda \frac{\partial \hat{u}}{\partial \xi} \right] - \frac{\tau_s - \tau_b}{\sigma H} \right\} \tag{32}$$

1. It can be shown that these terms are essential in determining the residual circulation (Nihoul and Ronday, 1976b).

where

$$\underset{\sim}{\tau}_s = \left[\nu \frac{\partial \underset{\sim}{u}}{\partial \xi} \right]_{x_3 = \zeta} \quad (33)$$

is the wind stress (normalized with water density).

It is possible to find an analytical solution of eq.(32) giving $\hat{\underset{\sim}{u}}$ in terms of $\underset{\sim}{\tau}_s$, $\underset{\sim}{\tau}_b$ and their derivatives with respect to time ; the coefficients depending on the functions $s(\xi)$, $b(\xi)$ and $f_n(\xi)$ (n=1,2...) defined by (Nihoul, 1977)

$$s(\xi) = \int_0^{\xi} \frac{\eta}{\lambda(\eta)} \, d\eta \quad (34)$$

$$b(\xi) = \int_{\xi_0}^{\xi} \frac{1 - \eta}{\lambda(\eta)} \, d\eta \quad (35)$$

$$\frac{d}{d\xi} [\lambda \frac{df_n}{d\xi}] = - \alpha_n f_n \quad (36)$$

with

$$\int_0^1 f_n^2(\xi) \, d\xi = 1 \quad (37)$$

$$\lambda \frac{df_n}{d\xi} = 0 \quad \text{at} \quad \xi = 0 \quad \text{and} \quad \xi = 1 . \quad (38)$$

One should note here that, in the definition of $b(\xi)$, the lower limit of integration is not set equal to zero but to $\xi_0 = \frac{z_0}{H} \ll 1$ where z_0 is the "rugosity length". z_0 can be interpreted as the distance above the bottom where the velocity is conventionally set equal to zero, ignoring the intricated flow situation which occurs near the irregular sea floor and willing to parameterize its effect on the turbulent boundary layer as simply as possible. In the North Sea, the value of z_0 , which varies according to the nature of the bottom, is of the order of 10^{-3} m ($\ln \xi_0 \sim - 10$) [e.g. Nihoul and Ronday, 1976a].

Although $\xi_0 \ll 1$, it cannot be systematically put equal to zero because the linear variation of the vertical eddy viscosity near the bottom leads to a logarithmic velocity profile which is singular at $\xi = 0$. However, in the present description, the difficulty exists only for the function b and the eigenfunction $f(\xi)$ may be determined on the interval $[0,\xi]$.

In a shallow tidal sea like the North Sea, it is readily seen, comparing the orders of magnitude of the different terms, that one obtains a very good approximation with only the first two terms in the series expansion of $\hat{u}$, i.e. (Nihoul, 1977)

$$\hat{u} = v_s[s(\xi) - \bar{s}] + v_b[b(\xi) - \bar{b}]$$

$$- [\frac{s_1}{\alpha_1 \sigma} \dot{v}_s + \frac{b_1}{\alpha_1 \sigma} \dot{v}_b] f_1(\xi) \qquad (39)$$

where $\bar{s}$ and $\bar{b}$ are the depth-averages of s and b , s_1 and b_1 two numerical coefficients ($s_1 = \int_0^1 s\, f_1\, d\xi$; $b_1 = \int_0^1 b\, f_1\, d\xi$) ; α_1 the eigenvalue corresponding to $f_1(\xi)$ and where

$$v_s = \frac{\tau_s}{\sigma H} \quad ; \quad v_b = \frac{\tau_b}{\sigma H} \qquad (40);(41)$$

A dot denotes here a total derivative with respect to time

$[\, \dot{v}_s = \frac{dv_s}{dt} = \frac{\partial v_s}{\partial t} + f\, e_3 \wedge v_s$ and similarly for $v_b\,]$.

Knowing the function $\lambda(\xi)$, one can determine $\hat{u}$ by eq.(39), $\hat{r}$ by eq.(21) and R by eq.(23).

Parameterization of the bottom stress

The functions $\hat{u}$, $\hat{r}$ and R depend on the vectors v_s , v_b and their derivatives. These can be determined by eqs.(27), (29), (40) and (41) from τ_s and τ_b .

The surface stress τ_s can be calculated from atmospheric data, the bottom stress τ_b is not given and must be determined by the no-slip condition at the bottom, i.e.

$$\hat{\underset{\sim}{u}} = - \overline{\underset{\sim}{u}} \quad \text{at} \quad \xi = \xi_0 \tag{42}$$

Eq.(42) provides a differential equation for $\underset{\sim}{\tau}_b$ in terms of $\underset{\sim}{\tau}_s$ and $\overline{\underset{\sim}{u}}$.

In shallow tidal seas like the North Sea, the terms including $\dot{\underset{\sim}{v}}_s$ and $\dot{\underset{\sim}{v}}_b$ are generally negligible and can only play a part during a relatively short time, at tide reversal (Nihoul and Runfola, 1979). The dominant term is, in fact, the term containing the bottom stress. The effect of the wind stress appears as a first order correction and the "memory" effect involving the derivatives $\dot{\underset{\sim}{v}}_s$ and $\dot{\underset{\sim}{v}}_b$ as a second order correction. One thus has

$$\overline{\underset{\sim}{u}} \sim \overline{b}\, \underset{\sim}{v}_b \qquad \text{(zeroth order)} \tag{43}$$

$$\overline{\underset{\sim}{u}} \sim \overline{b}\, \underset{\sim}{v}_b + \overline{s}\, \underset{\sim}{v}_s \qquad \text{(first order)} \tag{44}$$

$$\overline{\underset{\sim}{u}} \sim \overline{b}\, \underset{\sim}{v}_b + \overline{s}\, \underset{\sim}{v}_s + (s_1 \dot{\underset{\sim}{v}}_s + b_1 \dot{\underset{\sim}{v}}_b)\, \frac{f_{1,0}}{\alpha_1 \sigma} \qquad \text{(second order)} \tag{45}$$

Eq.(43) yields the well-known semi-empirical quadratic bottom friction law. Indeed, combining eqs.(27), (29) and (43), one finds

$$\|\underset{\sim}{\tau}_b\| \sim \frac{\sigma H}{\overline{b}} \|\overline{\underset{\sim}{u}}\| \sim \left(\frac{\sigma H}{k}\right)^2 \quad \Rightarrow \quad \sigma H \sim \frac{k^2}{\overline{b}} \|\overline{\underset{\sim}{u}}\| \tag{46}$$

i.e.

$$\underset{\sim}{\tau}_b = \frac{k^2}{\overline{b}^2} \|\overline{\underset{\sim}{u}}\|\, \overline{\underset{\sim}{u}} \tag{47}$$

$\frac{k^2}{\overline{b}^2}$ is the so-called "drag coefficient".

At the first order, one gets another classical formula (e.g. Groen and Groves, 1966; Nihoul, 1975) :

$$\underset{\sim}{\tau}_b = \frac{k^2}{\overline{b}^2} \|\overline{\underset{\sim}{u}}\|\, \overline{\underset{\sim}{u}} - \frac{\overline{s}}{\overline{b}}\, \underset{\sim}{\tau}_s \tag{48}$$

The second order parameterization is better understood if the last term in the right-hand side of eq. (45) is eliminated using eqs (32), (34), (35),

(36) and (39). One has, indeed

$$\frac{\partial \bar{\underline{u}}}{\partial t} + f\, \underline{e}_3 \wedge \bar{\underline{u}} = - \sigma \left[\frac{\partial}{\partial \xi} (\lambda \frac{\partial \hat{\underline{u}}}{\partial \xi})\right]_{x_3 = -h} + \frac{\underline{\tau}_s - \underline{\tau}_b}{H}$$

$$\sim - (s_1 \dot{\underline{v}}_s + b_1 \dot{\underline{v}}_b)\, f_{1,0} \sim - \alpha_1 \sigma\, (\bar{\underline{u}} - \bar{s}\, \underline{v}_s - \bar{b}\, \underline{v}_b)$$

i.e., using (46) to estimate σH ,

$$\tau_b \sim \frac{k^2}{\bar{b}^2} \|\bar{\underline{u}}\| \bar{\underline{u}} - \frac{\bar{s}}{\bar{b}} \underline{\tau}_s + \frac{H}{\alpha_1 \bar{b}} \left(\frac{\partial \bar{\underline{u}}}{\partial t} + f\, \underline{e}_3 \wedge \bar{\underline{u}}\right)$$

With a typical drag coefficient of the order of $2\ 10^{-3}$ (e.g. Nihoul and Ronday, 1976), one finds, using characteristic values for the North Sea,

$$\frac{k^2}{\bar{b}^2} \|\bar{\underline{u}}\| \bar{\underline{u}} \sim 0(2\ 10^{-3}\ \bar{u}^2) ,$$

$$\frac{H}{\alpha_1 \bar{b}} \frac{\partial \bar{\underline{u}}}{\partial t} \sim \frac{H}{\alpha_1 \bar{b}} f\, \underline{e}_3 \wedge \bar{\underline{u}} \sim 0(10^{-4}\ \bar{u}) .$$

Thus, the "acceleration" terms containing $\frac{\partial \bar{\underline{u}}}{\partial t}$ and $f\, \underline{e}_3 \wedge \bar{\underline{u}}$ (i.e. the terms arising from the time derivatives of $\underline{v}_s$ and $\underline{v}_b$) are not expected to play an important role except perhaps during relatively short periods of weak currents (when tides reverse, for instance). The total effect of the acceleration terms depends really on the local conditions. Obviously, (fig. 1) if the current velocity vector rotates clockwise during a tidal

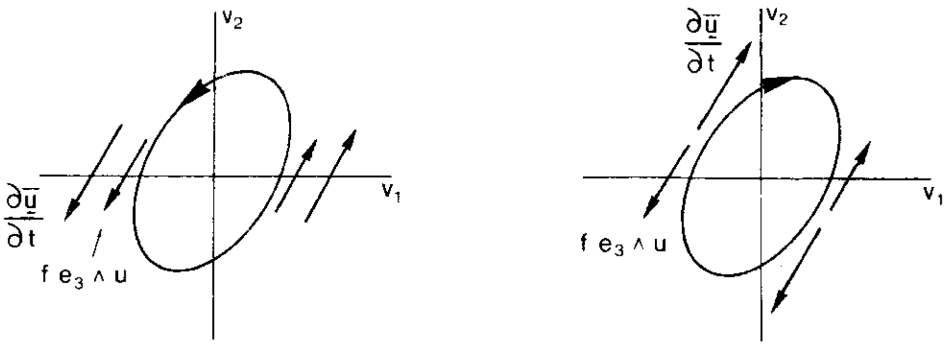

fig. 1.

period, the two terms tend to oppose each other and their global contribution may remain always very small. On the other hand, if the current velocity vector rotates counter-clockwise (as it is often the case in the Southern Bight) the two terms reinforce each other and have a definite - although limited - effect on the velocity profile and the related relationship between the bottom stress and the mean velocity (fig.2, fig. 3).

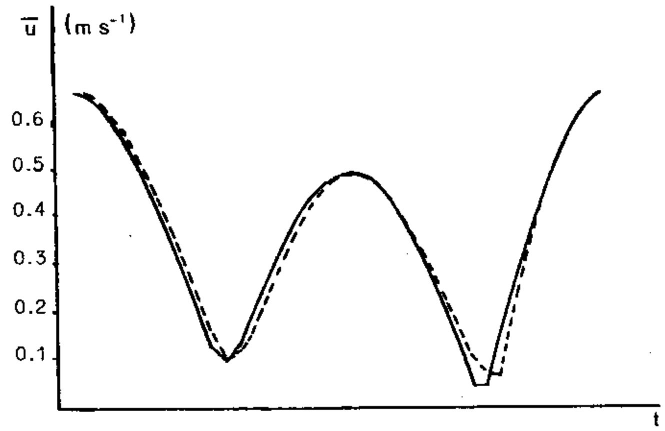

fig. 2.

Amplitude of the mean velocity

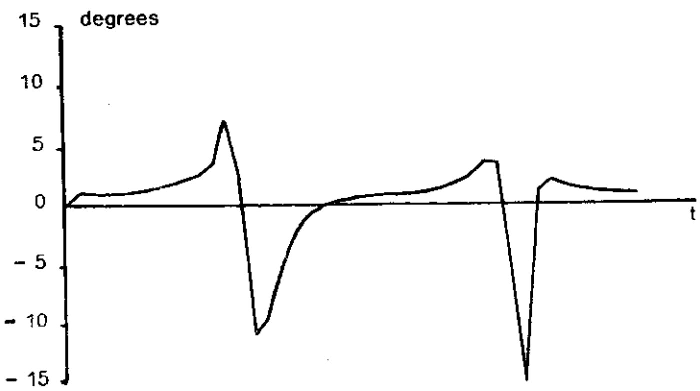

fig. 3.

Phase of the mean velocity

Comparison between the mean velocity $\overline{u}$ computed at the test point 52°30' N , 3°50' E in the North Sea by a depth-averaged two-dimensional model using an algebraic parameterization of τ_b (full line) and by the tree-dimensional model subject to the condition of zero velocity at the bottom (dash line) [Nihoul and Runfola, 1979].

Most hydrodynamic models of shallow seas restrict attention to the determination of the depth-mean velocity $\bar{\underset{\sim}{u}}$. The two-dimensional time dependent evolution equation for $\bar{\underset{\sim}{u}}$ is obtained from eq.(24) by integration over depth. It includes the surface elevation ζ and the bottom stress $\underset{\sim}{\tau}_b$ and thus constitutes with eq.(15) and eq.(43) (or 44) a closed system for the depth-averaged circulation (e.g. Nihoul, 1975).

The determination of the vertical velocity profile can be carried simultaneously using eq.(39).

One can go a step further and devise a three-dimensional model based on the depth-averaged equation, the local depth-dependent equation for the velocity deviation $\hat{\underset{\sim}{u}}$ and the refined parameterization of the bottom stress given by eq.(49) ; the acceleration corrections being taken into account when required in the numerical calculation. (Nihoul and Runfola, 1979).

The model gives H, $\bar{\underset{\sim}{u}}$, $\underset{\sim}{\tau}_b$, $\hat{\underset{\sim}{u}}$, $\hat{\underset{\sim}{r}}$ and $\underset{\approx}{R}$.

Substituting in eq.(16) one obtains explicitly the dispersion equation for the mean concentration $\bar{c}$.

Coefficients of the shear effect dispersion

Shear effect dispersion is described by eq.(16) where Σ is given, in terms of the functions $\hat{\underset{\sim}{r}}$ and μ, by eqs.(22) and (23). The functions $\hat{\underset{\sim}{r}}$ and μ can be determined by eqs.(21), (25), (31) and (39) provided the function λ is known. Thus the parameterization of the shear effect - as well as the determination of the vertical profile of velocity - reduces to the choice of a single scalar function.

The function

$$\lambda = \xi(1 - 0.5\ \xi) \tag{50}$$

appears to cover a wide range of situations in the North Sea and other shallow tidal seas (e.g. Bowden, 1965 ; Nihoul, 1977).

In this case, the eigenfunctions and the eigenvalues of eqs.(36), (37) and (38) are given by

$$f_n = (4n + 1)^{1/2} \quad P_{2n}(\xi - 1) \tag{51}$$

$$\alpha_n = n(2n + 1) \tag{52}$$

where P_{2n} is the Legendre polynomial of even order 2n.

Integrating eq.(39), one obtains, in this case,

$$\hat{\underset{\sim}{r}} = \underset{\sim}{v}_s \; S(\xi) + \underset{\sim}{v}_b \; B(\xi) + \frac{\dot{\underset{\sim}{v}}_s + 2\dot{\underset{\sim}{v}}_b}{\sigma} F(\xi) \qquad (53)$$

with

$$S(\xi) = 4 \ln 2(\xi - 1) + 2(2 - \xi)\ln(2 - \xi) \qquad (54)$$

$$B(\xi) = - 2 \ln 2(\xi - 1) + \xi \ln \xi -(2 - \xi)\ln(2 - \xi) \qquad (55)$$

$$F(\xi) = \frac{5}{36} (\xi^3 - 3 \xi^2 + 2 \xi) \qquad (56)$$

The shear effect diffusivity tensor can then be written, using eqs.(25) and (31)

$$\underset{=}{R} = \int_{\xi_0}^{1} \frac{\hat{\underset{\sim}{r}} \; \hat{\underset{\sim}{r}}}{\sigma\lambda} \, d\xi$$

$$= \frac{\gamma_{ss}}{\sigma} \underset{\sim}{v}_s \underset{\sim}{v}_s + \frac{\gamma_{sb}}{\sigma} (\underset{\sim}{v}_s \underset{\sim}{v}_b + \underset{\sim}{v}_b \underset{\sim}{v}_s) + \frac{\gamma_{bb}}{\sigma} \underset{\sim}{v}_b \underset{\sim}{v}_b$$

$$+ \frac{\gamma_{sf}}{\sigma^2} (\underset{\sim}{v}_s \dot{\underset{\sim}{v}}_s + 2 \underset{\sim}{v}_s \dot{\underset{\sim}{v}}_b + \dot{\underset{\sim}{v}}_s \underset{\sim}{v}_s + 2 \dot{\underset{\sim}{v}}_b \underset{\sim}{v}_s) \qquad (57)$$

$$+ \frac{\gamma_{bf}}{\sigma^2} (\underset{\sim}{v}_b \dot{\underset{\sim}{v}}_s + 2 \underset{\sim}{v}_b \dot{\underset{\sim}{v}}_b + \dot{\underset{\sim}{v}}_s \underset{\sim}{v}_b + 2 \dot{\underset{\sim}{v}}_b \underset{\sim}{v}_b)$$

$$+ \frac{\gamma_{ff}}{\sigma^3} (\dot{\underset{\sim}{v}}_s + 2 \dot{\underset{\sim}{v}}_b)(\dot{\underset{\sim}{v}}_s + 2 \dot{\underset{\sim}{v}}_b)$$

with

$$\gamma_{ss} = \int_{\xi_0}^{1} \frac{S^2}{\lambda} \, d\xi \sim 0.048 \qquad \gamma_{sf} = \int_{\xi_0}^{1} \frac{SF}{\lambda} \, d\xi \sim - 0.015 \qquad (58),(59)$$

$$\gamma_{sb} = \int_{\xi_0}^{1} \frac{SB}{\lambda} \, d\xi \sim 0.090 \qquad \gamma_{bf} = \int_{\xi_0}^{1} \frac{BF}{\lambda} \, d\xi \sim - 0.031 \qquad (60),(61)$$

$$\gamma_{bb} = \int_{\xi_0}^{1} \frac{B^2}{\lambda} \, d\xi \sim 0.196 \qquad \gamma_{ff} = \int_{\xi_0}^{1} \frac{F^2}{\lambda} \, d\xi \sim 0.005 \qquad (62),(63)$$

Application to the Southern Bight of the North Sea

In the Southern Bight of the North Sea, the depth is small and the bottom stress $\underset{\sim}{\tau}_b$, maintained by bottom friction of tidal currents, wind induced currents and residual currents is always fairly important. One can estimate that, in general, the characteristic time σ^{-1} is one order of magnitude larger than the characteristic time of variation of $\underset{\sim}{v}_s$ and $\underset{\sim}{v}_b$ (Nihoul and Ronday, 1976 ; Nihoul, 1977).

The terms of eq.(57) which contain the derivatives $\dot{\underset{\sim}{v}}_s$ and $\dot{\underset{\sim}{v}}_b$ - the coefficients of which are already smaller than the others - may then be neglected.

The shear effect diffusivity tensor reduces then to

$$\underset{\approx}{R} = \frac{H}{\|\underset{\sim}{v}_b\|} [\beta_1 \underset{\sim}{v}_b \underset{\sim}{v}_b + \beta_2 (\underset{\sim}{v}_s \underset{\sim}{v}_b + \underset{\sim}{v}_b \underset{\sim}{v}_s) + \beta_3 \underset{\sim}{v}_s \underset{\sim}{v}_s] \tag{64}$$

with

$$\beta_1 \sim 1.2 \quad ; \quad \beta_2 \sim 0.6 \quad ; \quad \beta_3 \sim 0.3 \qquad (65)(66)(67)$$

In weak wind conditions ($\underset{\sim}{v}_b \leqslant 10^{-2}\ \bar{\underset{\sim}{u}}$), the first term in the bracket is largely dominant and, using eq.(43), one obtains, with a good approximation

$$\underset{\approx}{R} = \alpha \frac{H}{\bar{u}} \bar{\underset{\sim}{u}}\, \bar{\underset{\sim}{u}} \tag{68}$$

$$\Sigma = H^{-1} \underset{\sim}{\nabla} . [\alpha \frac{H^2}{\bar{u}} \bar{\underset{\sim}{u}} (\bar{\underset{\sim}{u}} . \underset{\sim}{\nabla} \bar{c})] \tag{69}$$

with

$$\alpha \sim 0.14 \tag{70}$$

However, in weak wind conditions, the approximation which consists in neglecting the derivatives $\dot{\underset{\sim}{v}}$ and $\dot{\underset{\sim}{v}}$ is less justified and, furthermore, one may question the validity of eq.(50). If the wind is too weak to maintain turbulence in the sub-surface layer, one may expect, in some cases, a turbulent diffusivity which, instead of growing continuously from the bottom to the surface, instead passes through a maximum at some intermediate depth to decrease afterwards to a smaller surface value. This type of behaviour is described by the family of curves

$$\lambda = \xi(1 - \delta\xi) \tag{71}$$

Eq.(50) corresponds to the case $\delta = 0.5$. Values of δ from 0.5 to 1 correspond to lower intensity turbulence in the surface layer and the limiting value $\delta = 1$ would correspond to the case of an ice cover and the existence, below the surface, of a logarithmic boundary layer analogous to the bottom boundary layer.

In the Southern Bight of the North Sea, it is reasonable to assume that δ does not differ significantly from 0.5 and, in any case, never reaches extreme values close to 1. Nevertheless, to estimate the maximum error one can make on α , it is interesting to compute the coefficients β_1, β_2 and β_3 for some very different values of δ.

One finds

δ	0.5	0.7	0.9
β_1	1.2	1.5	2
β_2	0.6	0.8	1.3
β_3	0.3	0.5	1
α	0.14	0.17	0.23

The increase of the coefficients β_1, β_2, β_3 and α with δ is obviously associated with more important variations of u over depth i.e. with larger values of $\hat{u}$.

One should note also that the existence of a vertical stratification, even a weak one, reduces the turbulent diffusivity ($\mu = \eta\nu$ with $\eta < 1$) and contributes similarly to increase the value of α (e.g. Bowden, 1965).

In the Southern Bight of the North Sea, eventual modifications of the magnitude ($\eta < 1$) or of the form ($\delta > 0.5$) of the turbulent diffusivity are not likely to be very important and eq.(68) can presumably be used with $\alpha = 0.14$ or some slightly higher value obtained by calibration of the model with the observations.

Vertical concentration profile

When $\bar{c}$ has been calculated, it is possible to compute the deviation $\hat{c}$ by eq. (20). Changing variable to ξ and using eqs. (25) and (31), one gets

$$\frac{\partial \hat{c}}{\partial \xi} = \frac{\hat{\underline{r}}}{\lambda} \cdot \frac{\underline{\nabla} \bar{c}}{\sigma} \tag{72}$$

with, from eq. (9),

$$\int_0^1 \hat{c} \; d\xi = 0 \; . \tag{73}$$

Restricting attention to the dominant terms, one finds

$$\hat{c}(\xi) = [H(\xi) \; \underline{v}_s + G(\xi) \; \underline{v}_b] \cdot \frac{\underline{\nabla} \bar{c}}{\sigma}$$

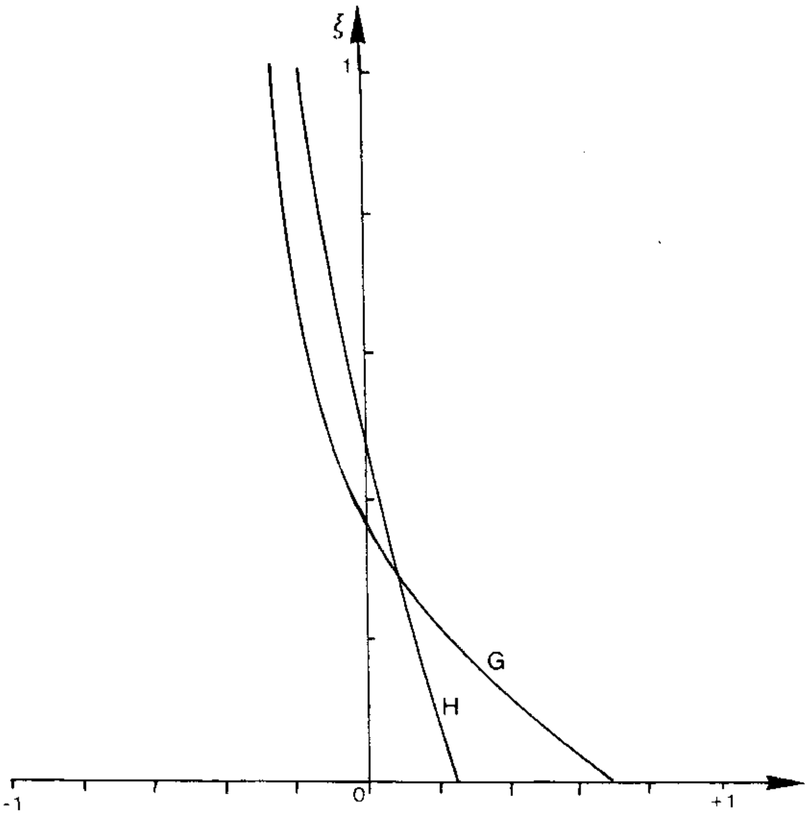

fig. 4.

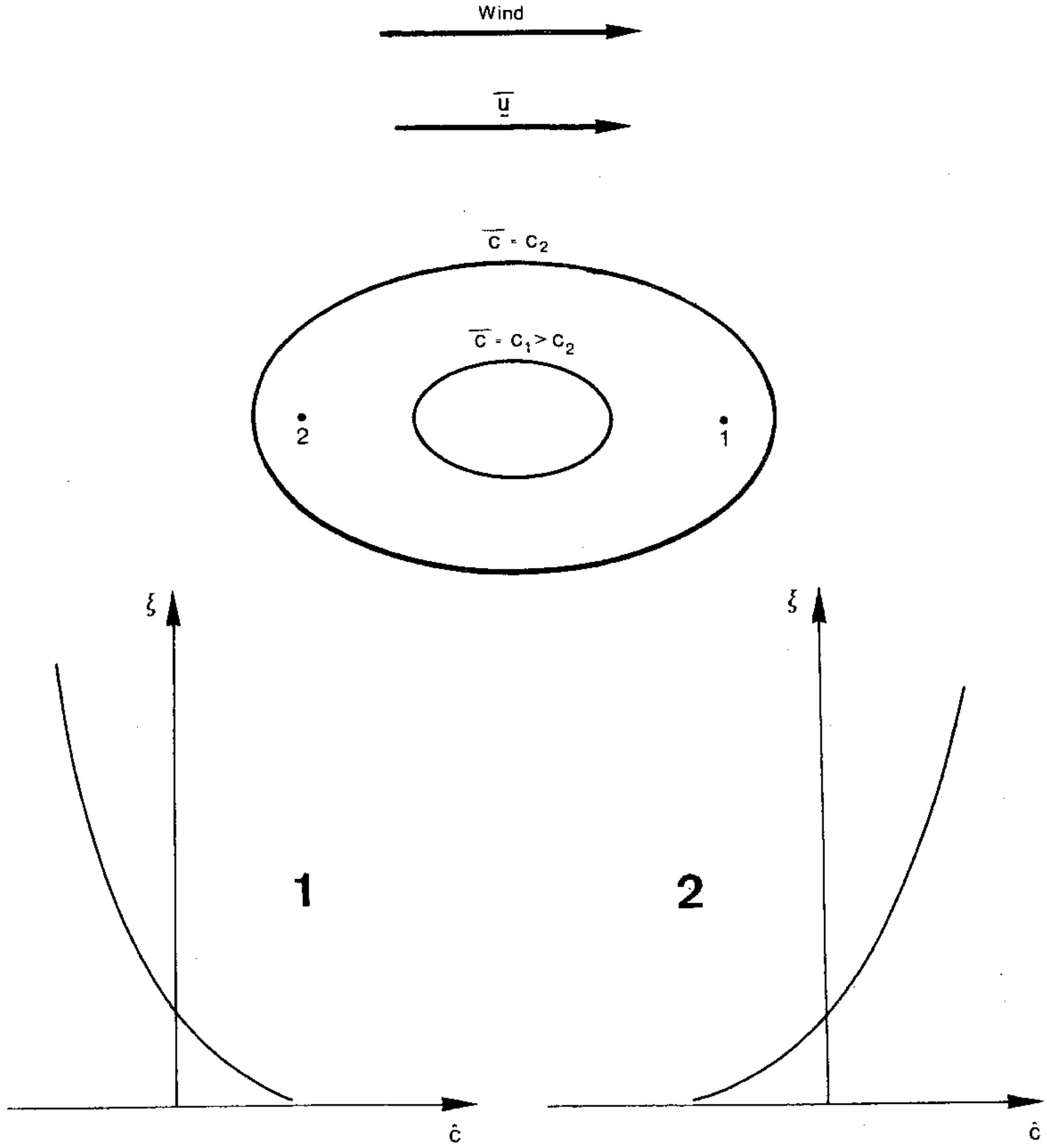

fig. 5.

where

$$H(\xi) = 4 \{P(\xi) + \ln (2-\xi) \ln \frac{\xi}{4} + \ln 2 (4 \ln 2 - 1 - \ln \xi) - 2\} \tag{75}$$

$$G(\xi) = -2 \{2 L_2(\frac{1}{2}) + \ln \frac{2-\xi}{2} \ln \frac{\xi}{2} + \ln (2) \ln (2) + 2\} \tag{76}$$

and

$$P(\xi) = L_2(\frac{\xi}{2}) + 2\, L_2(\frac{1}{2}) - \overline{L}_2 \tag{77}$$

$$L_2(x) = \text{Dilo}\,(1-x) = \sum_{\nu=1}^{\infty} (-1)^{\nu} \frac{(x-1)^{\nu}}{\nu^2} \tag{78}$$

The functions H and G are shown in fig.4. They are both negative near the surface and positive near the bottom. This is what one should expect from a physical point of view. Higher velocities near the surface carry water masses farther. If this transport is directed towards increasing mean concentrations the corresponding inflow of lower concentration fluid decreases the local concentration below the mean value $\bar{c}$. If the transport in the upper layer is directed towards decreasing mean concentrations, the corresponding inflow of high concentration fluid increases the local concentration above the mean value $\bar{c}$. The opposite situation occurs near the bottom. This is illustrated in fig. 5 showing the concentration profiles at two points situated downstream and downwind and respectively upstream and upwind on the same isoconcentration curve following a dumping.

The combination of eqs.(16) and (42) with a three-dimensional hydrodynamic model provides a three-dimensional dispersion model for the calculation of the evolution with time and the spatial - horizontal and vertical - distribution of any passive buoyant marine constituant.

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Air-sea interactions

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Introduction

Of late years, the investigation of air-sea interactions and the study of their effect on the dynamics of the ocean and the atmosphere have gained considerable importance.

Much work is at the moment being devoted to the measurement and parameterization of the different transfers occuring at the air-water interface. These consist mainly of fluxes of mass (water vapour,gases, pollutants), heat (short and long wave radiation, latent and sensible heat), momentum and mechanical energy.

At the local scale, these fluxes act as boundary conditions on the oceanic and atmospheric systems. They supply almost all the energy for circulations and turbulence in the ocean. They determine the state of the sea and of the water immediately below the interface. (A very important part of all the solar radiation absorbed below the bottom of the atmosphere is stored temporarily in the upper layers of the ocean). This portion of the sea being most under their influence is also the place where a very important biological activity occurs, which constitutes a crucial link in the food chain in the sea. Besides, they determine the state of the atmospheric lower layers, their stability, and their temperature, moisture and velocity distributions.

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1. Aspirant F.N.R.S.

Moreover, these fluxes are also of primary importance at the large scale : the climate of the earth is a very intricate system involving land, sea and air. As constant transfers occur continuously between the different components, the dynamics of the climate must take the oceanic and atmospheric dynamics into account, with all their interactions.

In this paper, we will present some of the work we are at present undertaking in this field.

In a first paragraph, the radiative and turbulent energy fluxes will be detailed ; several methods of measurement we are using will be described and compared, and the first experimental results will be presented.

In a second paragraph, some features of the atmospheric boundary layer and the ocean mixed layer and thermocline will be depicted. The general equations governing both phenomena will be introduced and different modelling approaches we are using will be described.

A last paragraph will introduce a large scale air-sea interaction problem and present some preliminary results: the ocean dynamics of the Gulf of Guinea (laying stress on the upwelling problem), and its possible connection with the climate of the Sahel area.

1.- Energy transfers and their measurement

1.1.- THE BASIC ENERGY TRANSFERS

A general view of the various energy fluxes involved in air-sea interactions is shown in figure 1. The two broad classes of exchanges are clearly depicted : radiative and turbulent.

1.1.1.- Radiative transfers

The basic source of energy is the sun. Its light undergoes many transformations before reaching the sea level. Diffusion and absorption by atmospheric molecules and clouds may reduce in a considerable way the energy arriving at the surface. Some light is also reflected by the sea surface (albedo), so that the remaining energy really absorbed in the upper layers of the ocean may vary within wide limits according to the season, time of the day and cloudiness.

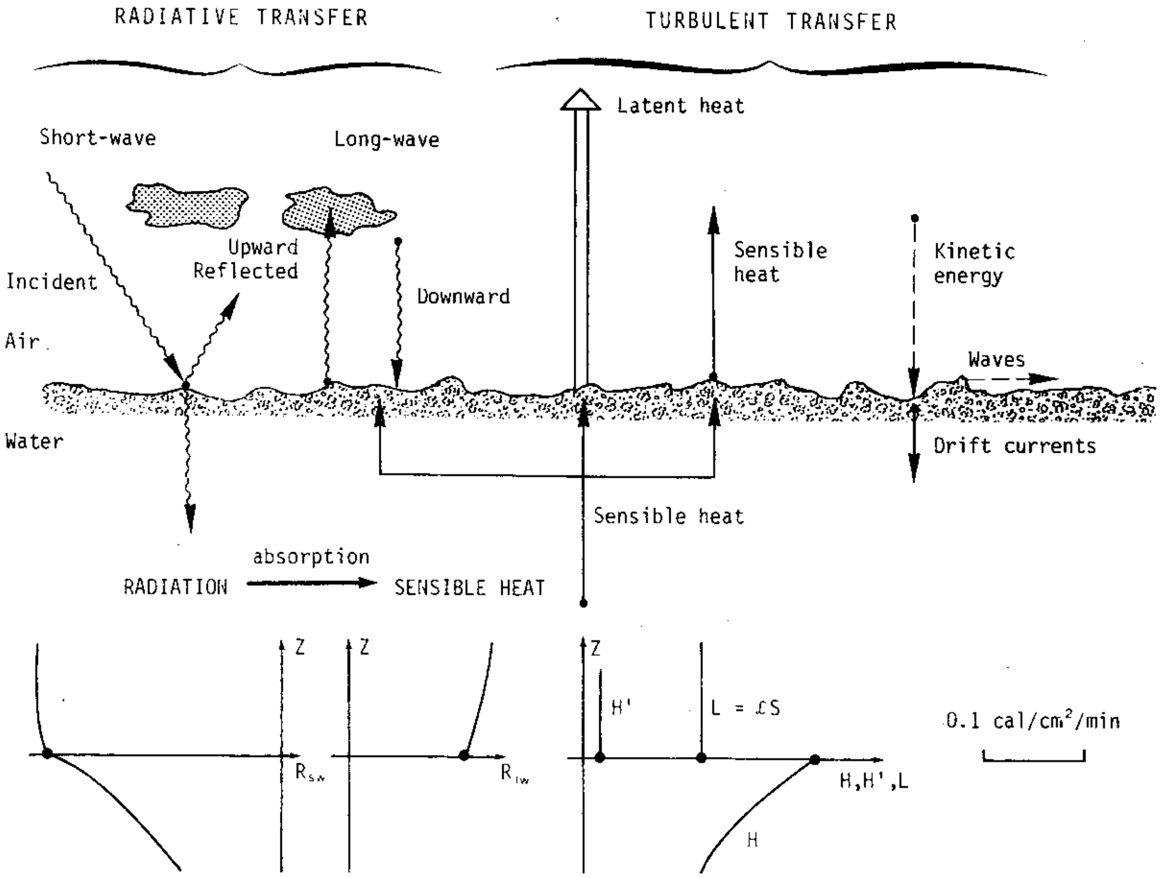


fig. 1.

Schematic display of energy transfers and transformations in the vicinity of the ocean-atmosphere interface, and the corresponding upward fluxes, assuming a steady-state situation.

In the infra-red part of the spectrum, a more complex equilibrium is reached between thermal emission from the sea surface and from the atmosphere above it. This is due to the high absorptive power of water vapour, of course present in large quantities in the marine atmosphere.

1.1.2.- Turbulent transfers

Friction on the sea surface generates turbulence in the air, enabling a turbulent transfer of energy. Some quantity of heat is exchanged in this way between the sea surface and the air, but the biggest part of the energy is extracted from the sea surface by evaporation of water, thus by transfer of latent heat.

Some part of the mechanical energy of the air is also transferred to the ocean (waves, current, oceanic turbulence).

The oceanic medium, subject to all these atmospheric inputs responds in its own way. The characteristics of its surface are modified (temperature, roughness), and this can in its turn affect the lower atmosphere. This is the basic feed-back mechanism of air-sea interactions.

This brief survey of the main exchanges between the atmosphere and the ocean makes the complexity of the problem in its generality obvious. This is the reason why most of the modelling approaches have to divide the task into smaller ones.

1.1.3.- Energy balance at the sea level

If S_0 denotes the solar incoming radiation flux and α the albedo of the sea surface, the following relation states that the sum of all energy fluxes is equal to zero :

$$I_0 - I_L^{\uparrow} + I_L^{\downarrow} - H - E + W = 0 \qquad (1)$$

(a positive sign means a gain of energy for the surface)
where

$$I_0 = S_0 \ (1 - \alpha) \ ,$$

$I_L^{\uparrow}$ is the infra-red emission of the surface, $I_L^{\downarrow}$ is the infra-red emission from the atmosphere, H is the sensible heat flux (to the atmosphere), E is the latent heat flux (to the atmosphere), W is the sensible heat flux in the sea (positive upwards).

1.2.- THE ESTIMATION OF THE FLUXES

1.2.1.- Radiative transfers

Measurements of radiative fluxes are difficult at sea mainly because of sea spray. After a few days in normal conditions, the sensors are covered with a film of salt, and a close watch on the instruments must be kept to ensure enough accuracy in the measurements. For that reason, only solar radiative flux can be obtained in coastal stations without much trouble. The albedo of the sea surface can now be fairly well determined from tables of sea surface reflectivity as a function of sun height, cloud cover and sea state (Krauss, 1972). Because of the sea spray, radiative measurements at sea can only be performed during special compaigns when a constant survey of the equipment is possible. The global or net radiative fluxes (upwards-downwards, both visible and infra-red) can then be measured. The only problem is to keep the sensor horizontal and to take into account the solid angle under which the sensor "sees" the boat.

1.2.2.- Turbulent transfers

There is no simple and direct way of measuring the turbulent energy fluxes, and indirect methods must be used. We shall briefly describe here four methods used to determine the fluxes at the local scale. They are :

a) drag coefficient formulas

b) similarity and surface layer theory

c) direct measurement of fluctuation covariances

d) spectral methods

a) *Drag coefficient*

This is the simplest way of estimating the fluxes. The only quantities needed are the wind speed at some standard level (usually 10 m) above the sea surface, the air temperature and humidity at the same level, and the sea surface temperature.

In the turbulence theory, the fluxes are given by :

$$\tau = \rho_a u_*^2 \quad (2)$$

$$H = - \rho_a C_{p_a} u_* T_* \quad (3)$$

$$E = - \rho_a L u_* q_* \quad (4)$$

where τ is the surface stress or flux of momentum, H and E are the sensible and latent heat fluxes, ρ_a is the density of the air, C_{p_a} is the heat capacity of the air at constant pressure, L is the latent heat of vaporization of the water, u_* is the friction velocity in the air, T_* and q_* are the temperature and humidity scales of turbulent fluctuations.

The first drag coefficient C_D is defined as the square of the ratio of the friction velocity u_* to the wind speed V measured at the standard level, i.e. :

$$u_* = C_D^{\frac{1}{2}} V_{10} .$$

The other coefficients C_E and C_H can be introduced in a similar way. The drag formulas are thus :

$$\tau = \rho_a C_D V_{10}^2 \quad (5)$$

$$H = - \rho_a C_{p_a} C_H V_{10} (T_{10} - T_w) \quad (6)$$

$$E = - \rho_a L C_E V_{10} (q_{10} - q_w) \quad (7)$$

where T_{10} is the air temperature at 10 m height, q_{10} is the air humidity at 10 m height, T_w is the water temperature, q_w is the specific humidity of saturated air at the water temperature (this quantity is determined from T_w and the tables of thermodynamic properties of water vapour).

In fact, the drag coefficients C_D, C_E and C_H simply relate the measurable quantities to unmeasurable ones. These coefficients have been carefully determined in special measurement campaigns by a best fit to the observed flux data. Unfortunately, there is a wide variety of C_D in the literature. We present here one of the latest version reported by Friehe and Gibson (1978). If

$$\Delta T = T_w - T_{10} ,$$

$$C_D = 10^{-3} \times (0.63 + 0.066 V_{10}) \quad (8)$$

$$C_H = \begin{cases} 10^{-3} \times (2 + 0.97 V_{10} \Delta T) \\ 10^{-3} \times (1.46 V_{10} \Delta T) \end{cases} \quad \text{if} \quad V_{10} \Delta T \begin{cases} < 25 \text{ m s}^{-1} \text{ K} \\ > 25 \text{ m s}^{-1} \text{ K} \end{cases} \quad (9)$$

$$C = 10^{-3} \times 1.32 \quad (10)$$

(V_{10} in m s^{-1}, T in Kelvins).

As a rule, the C_D's are increasing with increasing wind speed, because of the increasing roughness length z_0 of the sea.

This is the simplest way of obtaining valuable estimates of the fluxes. However, a non negligible scatter of C_D values among different authors may question the general applicability of the method.

b) *Similarity theory – Surface layer formulas*

The original similarity theory of Monin and Obukov published in 1954 has been widely used recently since the determination of the functions by Businger (1973). The theory stipulates that the vertical non dimensionalised wind, temperature and humidity gradients are only function of a non dimensional height $\xi = z/L$, where L is the Monin-Obukov length scale, defined as :

$$L = \frac{\overline{T}\, u_*^2}{kgT_*} \tag{11}$$

g is the acceleration of gravity and $\overline{T}$ the mean temperature, k is the Von Karman constant (k = 0.35).

Thus

$$\begin{cases} \dfrac{kz}{u_*}\dfrac{\partial \overline{u}}{\partial z} = \phi_M(\xi) \\[2ex] \dfrac{kz}{T_*}\dfrac{\partial \overline{T}}{\partial z} = \phi_H(\xi) \\[2ex] \dfrac{kz}{q_*}\dfrac{\partial \overline{q}}{\partial z} = \phi_E(\xi) \end{cases} \tag{12}$$

The ϕ_i (i = M,H,E) have now a well known analytical form (Businger, 1973).

The relations (I.12) can be integrated from 0 (or z_0) to the standard level of measurement (usually 10 m), yielding the integrated form ψ_i of the ϕ_i. We have then :

$$
\begin{cases}
u_* = \dfrac{k\, V_{10}}{\ln \dfrac{z}{z_0} - \psi_M} \\[2ex]
T_* = \dfrac{k(T_{10} - T_w)}{0.74 \ln \dfrac{z}{z_0} - \psi_H} \\[2ex]
q_* = \dfrac{k(q_{10} - q_w)}{0.74 \ln \dfrac{z}{z_0} - \psi_E}
\end{cases}
\qquad (13)
$$

We obtain in this way an implicit set of equations that can be solved iteratively to have L , u_* , T_* and q_* from the observed V_{10} , T_{10} , T_w and q_w . The fluxes can then be obtained directly from equations (2) to (4). It is important to note that in order to use equations (13), an adequate value of z_0 must be determined independently, generally as a function of V_{10} .

c) *Direct method*

With fast response sensors and a fixed, stable frame of reference, the fluctuating eddies in the air may be resolved and the turbulent fluxes can be directly expressed as the covariances of the fluctuations of the vertical wind component and the other quantity. Thus :

$$
\begin{cases}
\tau = - \rho_a \, \overline{u'w'} \\[1ex]
H = \rho_a \, C_{p_a} \, \overline{T'w'} \\[1ex]
E = \rho_a \, L \, \overline{q'w'}
\end{cases}
\qquad (14)
$$

(Here an overbar denotes a time average over an adequate period, generally of the order of a few tens of minutes, and primes denote fluctuations around the mean).

Comparing (I.2 to I.5) with (I.14) leads immediately to :

$$\begin{cases} \overline{u'w'} = - u_*^2 \\ \overline{T'w'} = - u_* T_* \\ \overline{q'w'} = - u_* q_* \end{cases} \qquad (15)$$

which can be used to define the velocity, temperature and humidity scales u_* , T_* and q_* .

This technique may be considered as an absolute measurement of the fluxes (a reference one), but it needs sensors able to respond to frequencies up to a few Hz in order to pick up the portion of the flux due to small eddies. In the atmosphere, this implies a sampling frequency of the order of 10 Hz.

The other important point is that the verticality of the sensor must be accurately set, and must remain unchanged. Equations (I.14) show that w' is the leading factor. Even a very small deviation of the sensor from the vertical should induce the effect of u or v in the measured w (since generally $\overline{u} \simeq \overline{v} \gg \overline{w}$). This implies the use of a fixed rigid support for the instrument (a pile e.g.) and excludes any simple system (as usual buoys or boats).

d) *Spectral method*

This promising method will be briefly outlined here. For details, one can refer to Champagne et al. (1977).

In the inertial subrange of turbulence, the spectrum of wind fluctuations follows the Kolmogorov's law :

$$S_u(n) = \alpha_1 \, u^{\frac{2}{3}} \, \varepsilon^{\frac{2}{3}} \, n^{-\frac{5}{3}} \qquad (16)$$

where α_1 is a constant $\simeq 0.55$, ε is the dissipation rate, n is the frequency measured at a fixed point in space.

In neutral conditions, the wind profile follows the logarithmic law :

$$\frac{\partial u}{\partial z} = \frac{u_*}{kz} \; .$$

As the turbulent energy balance in the stationary case is

$$-\overline{u'w'}\ \frac{\partial u}{\partial z} = \varepsilon \ ,$$

we have

$$u_* = (k\,\varepsilon\,z)^{\frac{1}{3}} \ .$$

In the non-neutral case, it can be shown that this relation becomes :

$$u_* = \left[\frac{k\ \varepsilon\ z}{\left[1 + 0.5\ \left|\frac{z}{L}\right|^{2/3}\right]^{3/2}} \right]^{1/3} \qquad (17)$$

As ε can be found from (16), equation (2) gives an evaluation of the momentum flux if L is known.

A similar procedure can be followed for temperature and humidity : the spectrum of temperature fluctuations follows the law

$$S_\theta(n) = \beta_\theta\ \chi_\theta\ u^{\frac{2}{3}}\ \varepsilon^{-\frac{1}{3}}\ n^{-\frac{5}{3}} \qquad (18)$$

where β_θ is a constant $\simeq 0.4$, χ_θ is the dissipation rate for temperature fluctuations (to be determined). The use of a similar hypothesis for the temperature profile leads to the following expression for the temperature scale :

$$T_* = -\left[\frac{k\,z\,\chi_\theta}{2u_*\phi_H}\right]^{\frac{1}{2}}$$

where ϕ_H is the similarity function for heat.

The heat flux is then found by equation (3).

A quite similar relation is also established for q_*, but at the present time, no experimental confirmation exists for it.

e) *Comparison of the methods*

Table 1 summarizes the needs and advantages of each method.

The last two methods require instruments capable of measuring frequencies over 1 Hz in the fluctuations. The ideal instrument for this purpose is the sonic anemometer. The Gill anemometer we have can also be used, if it is not too close to the sea surface.

Table 1

Comparison of the methods used to determine the turbulent fluxes

Method	Data needed	Instrument needed	Comments	Maximum expected error
A. Drag coefficient	T_{10}, T_w, V_{10}, q_{10} + choice of drag coefficients formulas	Simple instruments measuring mean values Use of buoy possible.	Very fast method	± 40 %
B. Similarity Monin-Obukov	T_{10}, T_w, V_{10}, q_{10} + estimated z_0		Iterative method	± 40 %
C. Direct covariance	u, v, w, T, q	Fast response instruments to measure fluctuating quantities. Fixed frame of reference (tower or mast fixed on bottom) Vertical must be accurately defined.	Basic technique of flux determination Difficult method at sea.	± 10 %
D. Spectral	u, T, q	Fast response instruments to measure fluctuating quantities. Fixed frame of reference not necessary. Vertical wind speed not needed. Use of buoy should be possible.		± 20 % (?)

1.3.- A FEW RESULTS

A buoy has been operated in common by the Institut Royal Météorologique (I.R.M.), the University of Liège, and the University of Louvain-La-Neuve, during two summer campaigns at STARESO, the oceanographic station of the University of Liège in Calvi, Corsica.

The available data are summarized in table 2.

Table 2

Data measured by the I.R.M. buoy in Calvi

Campaign	Period of recording	Measured parameters
Summer 1977	14-7-77 to 10-8-77 (with interruptions)	Wind speed, Air temperature } at 6 m height Sea surface temperature at - 2 m
Summer 1978	9-7-78 to 9-8-78	Wind speed, Wind direction, Air temperature } at 6 m height Wind speed, Air temperature } at 2 m height Water temperature at - 2 m

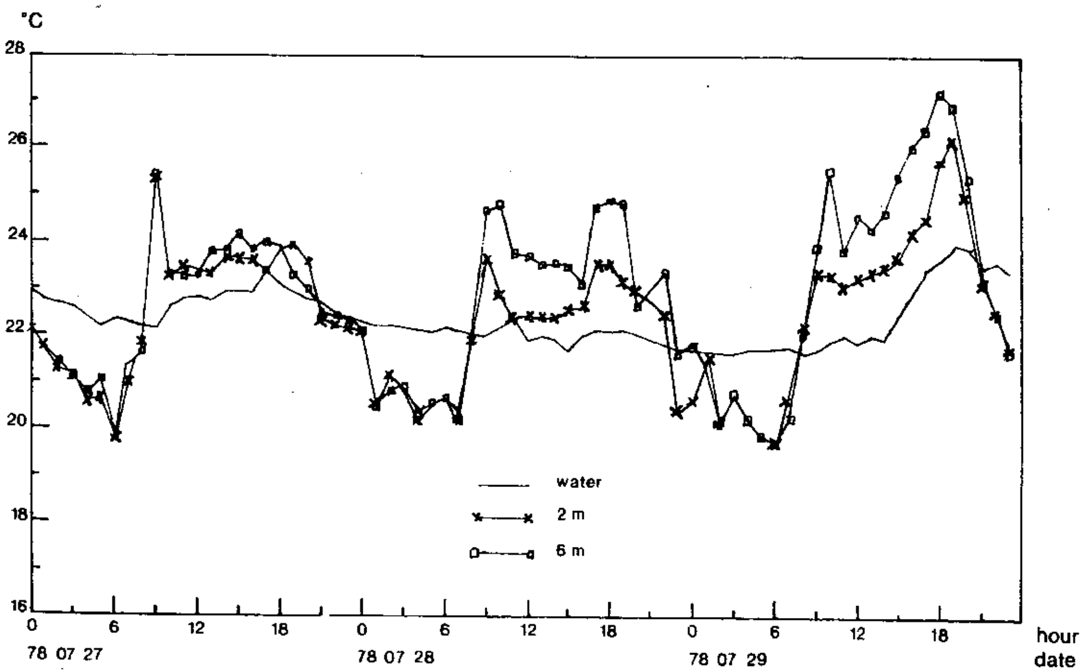


fig. 2.

Sample of observed temperatures in the Bay of Calvi with the I.R.M. buoy

Figures 2 and 3 show a sample of the measured data. Figure 4 shows the estimated fluxes for the 1978 summer campaign.

The reader may refer to the work of Clement (1979) for complete fluxes calculations and comparisons. The complete set of data is available at the I.R.M.

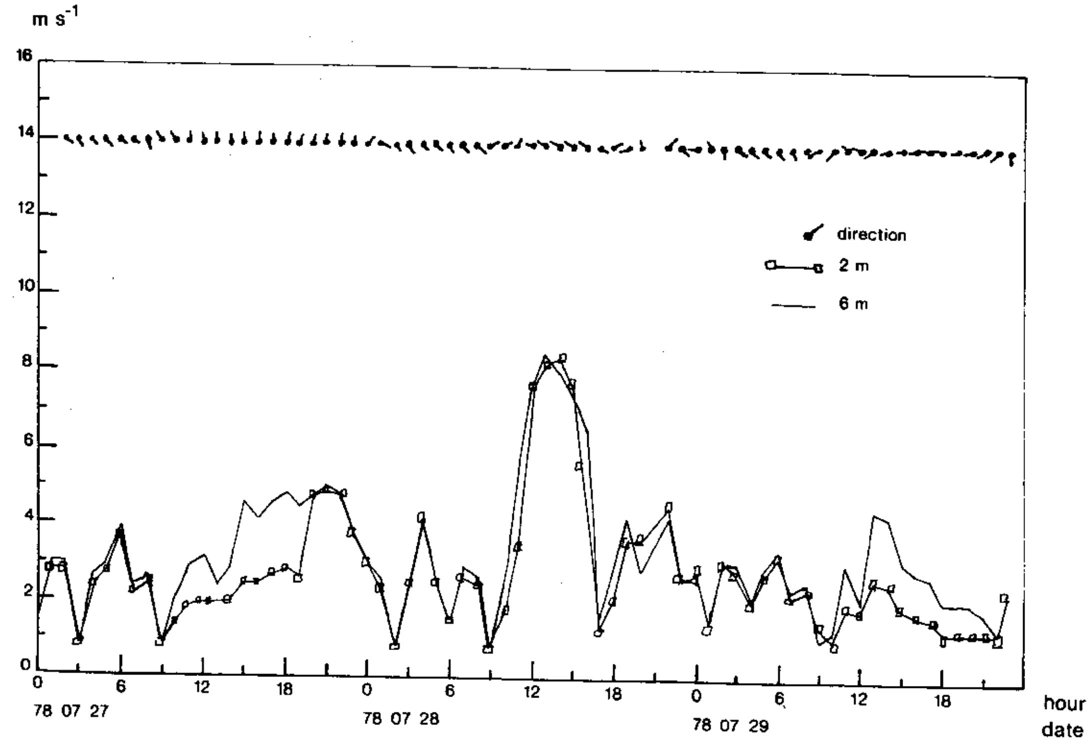


fig. 3.

Sample of observed wind speed and direction in the Bay of Calvi with the I.R.M. buoy

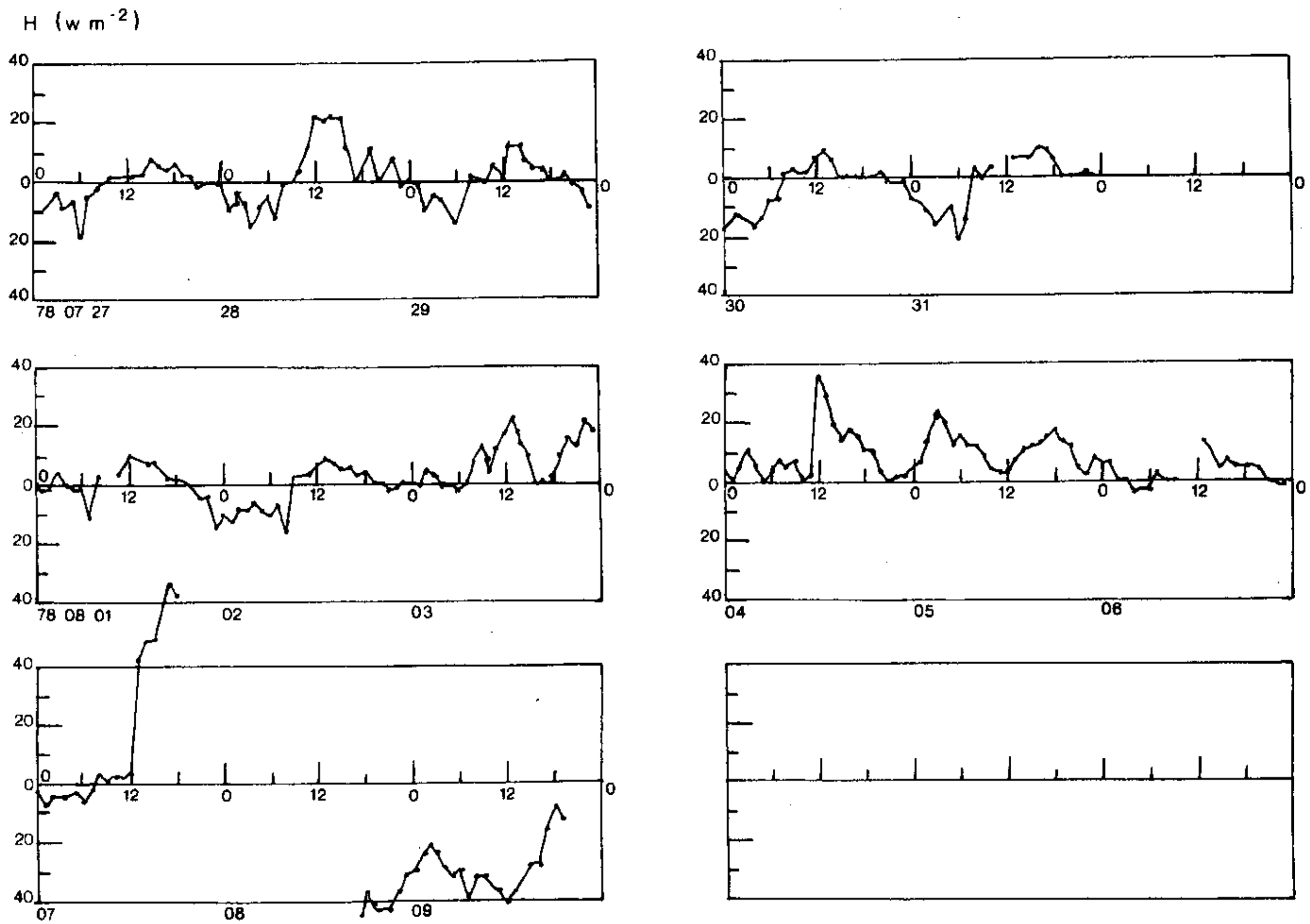


fig. 4.

Estimated heat fluxes from 27-07 to 09-08-1978

2.- The boundary layers in the atmosphere and the ocean

In the preceding section, several processes occuring close to the sea surface (at a few meters height) have been described in order to measure the turbulent fluxes at the interface.

As mentioned in the introduction, the fluxes act also as lower boundary conditions for the atmospheric boundary layer, i.e. the layer in which turbulent exchanges of momentum, heat and moisture take place. Typically, this layer is from 500 to 1500 meters high. Above this level, the friction of the sea surface is no more felt, and the air flux is normally non turbulent.

They act also as boundary inputs for the upper layers of the ocean. However, the ocean has a much larger thermic and dynamic inertia than the atmosphere, and very often exhibits an important stratification which acts as a barrier for the exchanges with lower water layers. This stratification is mainly due to heat and salinity.

Before starting with the equations used to model the atmospheric boundary layer and the upper layers of the ocean, it is interesting to have a look at the general characteristics of the ocean vertical thermal structure.

2.1.- MAIN FEATURES OF THE VERTICAL THERMAL STRUCTURE OF THE OCEAN

In this section, we shall not consider the recently discovered phenomenon of fine-scale stratification or microstructure in the ocean, although this could lead to a reconsideration of the modelling problem, but emphasize the coarser general structure of the oceanic waters. (Further informations about microstructure can be found in Fedorov, 1978).

In low and mid latitude areas, the mean vertical structure of the waters (disregarding annual or higher frequencies fluctuations) can be schematized by the existence of three different layers :

a) a surface layer, 50 to 200 m deep, where the temperature is close to its value at the surface ;

b) a layer in which the temperature decreases with depth and which extends itself below the first and down to 1000 m almost. This is the main or permanent thermocline.

c) deeper, the temperature gradient dies away : we are in the domain of the deep waters.

There are also some differences between the equatorial and mid-latitude areas : in the former ones, the waters are highly stratified and a well defined thermocline can be found ; in the latter ones, there is also a thermocline, but not so important and often deeper.

However, polar areas, many lakes and some enclosed seas (as the Mediterranean Sea) do not exhibit a permanent thermocline. This mean global structure is mainly the result of the large scale energetic exchanges with the atmosphere and the general oceanic circulation.

The layers close to the sea surface (0 - 100 m) undergo the influence of the exchanges with the atmosphere, and these have several well or less marked periodicities (seasonal, diurnal, synoptic,..). Heat fluxes depend locally on the season, hour of the day and atmospheric conditions. Mechanical energy transfers, depending mainly on the wind, can also exhibit a periodicity of a few days (synoptic, i.e. related to the development of atmospheric perturbation in our latitudes, e.g.). The advection can also take an important part locally.

In connection with the seasonal variations of the fluxes in mid and high latitude areas, a seasonal thermocline develops at the bottom of a layer often close to homogeneity in temperature. This thermocline appears in spring, develops in summer and dies away at the end of autumn. The temperature difference between the upper warm layers and the underlying fluid is more important at mid-latitude than in polar areas. In low latitude areas, the seasons are not so distinct, and there is almost no variation across the year.

Finally, diurnal or synoptic variations in the fluxes can also induce diurnal or transitory thermocline structures. In special circumstances, a succession of transitory thermoclines can be observed above the seasonal thermocline, related to the evolution of the fluxes in the preceding days.

It should again be noticed that polar areas, some lakes and enclosed seas as the Mediterranean Sea have only seasonal, diurnal and transitory thermoclines.

One important problem, and also one of the best documented, is the study of the effect of a gust of wind causing mixing in the upper layers and entrainment of the underlying fluid. The new mixed layer can enclose old thermoclines and even reach the seasonal thermocline and modify it.

Figure 5 shows an example of a well develop mixed layer due to a sudden rise in wind force. The profiles were taken at STARESO, Calvi, Corsica.

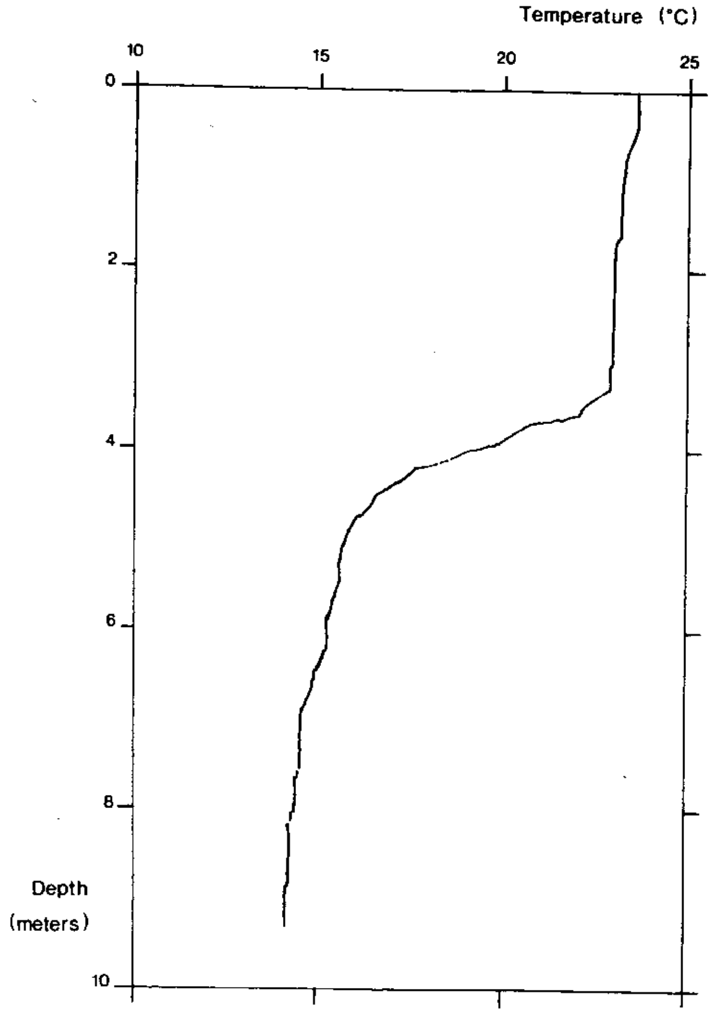


fig. 5.

Vertical structure of the water temperature off Calvi, Corsica, after the passage of a gust of wind (10 m/s) [18-07-77]

One of our present aims is to model the dynamics of the upper oceanic layers at the different time scales. If the question of the mixed layer deepening is well documented, its later evolution and restructuration still set unresolved experimental and theoretical problems.

2.2.- BOUNDARY LAYER MODELLING

The fundamental hypotheses used for the modelling of the atmospheric boundary layer and of the oceanic upper mixed layer and thermocline are the following :

1) The Boussinesq approximation.

In the ocean

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)] , \quad \rho_0 = \rho(S_0, T_0)$$

where T is the temperature and S the salinity. In the atmosphere, a similar formula is introduced, involving potential temperature and water-vapour content.

2) The parameters are decomposed into a mean value (denoted by an overbar) and turbulent fluctuations. The equations for the mean values of the parameters are considered, as well as the equation for turbulent kinetic energy.

Nowadays, second order closure models are being developed. These consider also the equations for all second order covariances and use closure hypotheses for these equations. However, they involve many computation problems and in a first time, we will restrict ourselves to the former, simpler but more direct approach.

3) Horizontal homogeneity is also assumed. This hypothesis is realistic in the marine atmospheric boundary layer, because of the absence of topography. In the ocean, it is also realistic if we consider areas far enough from coastal areas and bottom effects.

We get in this way :

$$\frac{\partial u}{\partial t} = f(v - v_g) - \frac{\partial}{\partial z} \overline{u'w'} \qquad (19)$$

$$\frac{\partial v}{\partial t} = - f(u - u_g) - \frac{\partial}{\partial z} \overline{v'w'} \qquad (20)$$

$$\frac{\partial T}{\partial t} = - \frac{\partial}{\partial z} \overline{w'T'} + \frac{R}{\rho_{0_w} c_{p_w}} \qquad \text{(ocean)}$$

or (21)

$$\frac{\partial \theta}{\partial t} = - \frac{\partial}{\partial z} \overline{w'\theta'} + \frac{R}{\rho_{0_a} c_{p_a}} \qquad \text{(atmosphere)}$$

$$\frac{\partial S}{\partial t} = - \frac{\partial}{\partial z} \overline{w'S'} \quad \text{(ocean)}$$

or

$$\frac{\partial q}{\partial t} = - \frac{\partial}{\partial z} \overline{w'q'} \quad \text{(atmosphere)} \tag{22}$$

$$\frac{\partial e}{\partial t} = - \overline{u'w'} \frac{\partial u}{\partial z} - \overline{v'w'} \frac{\partial v}{\partial z} + \overline{w'b'} - \frac{\partial}{\partial z} \left[\frac{\overline{p'w'}}{\rho_0} + \overline{\frac{u'^2 + v'^2 + w'^2}{2} w'} \right] - \varepsilon \tag{23}$$

The frame of reference is dextrorsum, with z positive upwards. The overbars have been omitted for mean speeds (u,v), temperature, or potential temperature in the atmosphere (T,θ), salinity (S), humidity (q) and turbulent kinetic energy (e). f is the Coriolis parameter and (u_g, v_g) the geostrophic wind or current. ρ_0 and c_p are related to the air, or the sea water, R is the divergence of the radiation flux, b' denotes the fluctuations of buoyancy b defined as $b = -\frac{\rho - \rho_0}{\rho_0} g$. An equation for buoyancy can be written instead of (21) and (22).

The right-hand side terms of the last equation have the following physical meaning :

$- \overline{u'w'} \frac{\partial u}{\partial z} - \overline{v'w'} \frac{\partial v}{\partial z}$ is the turbulence production due to shear in the mean wind or current ,

$\overline{b'w'}$ is a source or a sink of turbulent kinetic energy due to the buoyancy forces ,

$\frac{\partial}{\partial z} \left[\frac{\overline{p'w'}}{\rho_0} + \overline{\frac{u'^2 + v'^2 + w'^2}{2} w'} \right]$ is a transport term of turbulence,

ε is the turbulent kinetic energy dissipation (ε > 0).

Closure hypotheses have to be introduced now in order to solve the system. We also need initial and boundary conditions for the different variables.

We will examine the particular cases of the atmospheric and oceanic boundary layers.

2.2.1.- Atmospheric boundary layer

The turbulent diffusivities K_M , K_θ and K_q are introduced :

$$- \overline{u'w'} = K_M \frac{\partial u}{\partial z} \quad ; \quad - \overline{v'w'} = K_M \frac{\partial v}{\partial z} \tag{24}$$

$$- \overline{w'\theta'} = K_\theta \frac{\partial \theta}{\partial z} \tag{25}$$

$$- \overline{w'q'} = K_q \frac{\partial q}{\partial z} \tag{26}$$

In this case, if we neglect the divergence of the radiation flux, equations (19) to (22) become :

$$\frac{\partial u}{\partial t} = f(v - v_g) + \frac{\partial}{\partial z} (K_M \frac{\partial u}{\partial z}) \tag{27}$$

$$\frac{\partial v}{\partial t} = - f(u - u_g) + \frac{\partial}{\partial z} (K_M \frac{\partial v}{\partial z}) \tag{28}$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} (K_\theta \frac{\partial \theta}{\partial z}) \tag{29}$$

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial z} (K_q \frac{\partial q}{\partial z}) \tag{30}$$

These four equations can be solved numerically if initial profiles of u , v , θ and q are given as well as the temperature and humidity time evolution at the sea level. The K's are used as parameters. Many kinds of ABL models have been devised assuming various forms of $K(z)$ profiles. The interest reader can find a review of $K(z)$ specifications in Yu (1977), Coantic (1978) and Schayes (1979).

A not complicated and realistic K formulation is based on equation (II.5). The introduction of the diffusivities and several hypotheses lead to :

$$\frac{\partial e}{\partial r} = K_M \Bigg(\underbrace{[\frac{\partial u}{\partial z}]^2 + [\frac{\partial v}{\partial z}]^2}_{(a)} - \underbrace{1.35 \frac{g}{\theta} \frac{\partial \theta}{\partial z}}_{(b)} \Bigg) + \underbrace{1.2 \frac{\partial}{\partial z} [K_M \frac{\partial e}{\partial z}]}_{(c)} - \underbrace{\varepsilon}_{(d)} \tag{31}$$

where we find again the expressions for turbulent kinetic energy production by wind shear (a), production or removal due to buoyancy (b), diffusion (c) and dissipation (d).

The dissipation ε is frequently expressed as

$$\varepsilon = c\, e^{\frac{3}{2}}\, \ell \qquad \text{(c is a constant)}$$

and K_M is defined by

$$K_M = c\, \ell\, e^{\frac{1}{2}}$$

The set of equations is then closed if we specify the mixing length ℓ. Various formulations exist among which the one of Blackadar is frequently used :

$$\ell = \frac{kz}{1 + \frac{kz}{\lambda}} \quad \text{or} \quad \frac{1}{\ell} = \frac{1}{\lambda} + \frac{1}{kz}$$

where

$$\ell = 2.7 \quad 10^{-4} \left|\frac{U_g}{f}\right| \qquad (U_g \text{ is the geostrophic wind}).$$

Up to this point, the model fits mainly continental ABL simulation. On land, turbulent processes are dominating due to the normally large temperature oscillation of the night and day sequence. Large instabilities can develop during the day, triggering convection. On the other hand, over the sea, the daily temperature oscillation (at sea level) is very much smaller, and instability does not develop in daytime (unless advection of cold air over warm water occurs).

Therefore, radiative and evaporation phenomena are as important as turbulent ones over the marine ABL and we must take the radiative term into account in (29).

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[K_\theta \frac{\partial \theta}{\partial z}\right] + \frac{R}{\rho_{0_a}\, c_{p_a}} \qquad (32)$$

$$R = - \frac{\partial}{\partial z} (F^{\uparrow} - F^{\downarrow}) \qquad (33)$$

R is the heating rate due to the divergence of the radiation flux density. $F^{\uparrow}$ is the total radiative flux upwards, and $F^{\downarrow}$ the total radiative flux downwards at the considered level.

The F's are computed by integrating the absorptivity - emissivity of water vapour at all levels above or below the considered level. For example $F^{\downarrow}$ is given by :

$$F^{\downarrow}(z_1) = \int_{z_1}^{\infty} \sigma T^4 \frac{d\varepsilon(z_1,z)}{dz} dz \qquad (34)$$

where σ is the Stefan constant (black body) and ε is here the emissivity of water vapour between levels z_1 and z . A similar equation is used for $F^{\uparrow}$.

Such a radiative model should simulate the marine ABL in a reasonably good way as far as advective phenomena are not important.

Let us notice two points now :

- this model does not take any condensation of water vapour into account, i.e. the presence or formation of clouds above the BL (medium and high clouds) and in the BL (low clouds), which affect considerably the radiative balance of the system. These important problems have not yet found a good solution. They are often parameterized in a crude way in order to avoid very long computation time.

- if horizontal homogeneity cannot be assumed, one has to turn to a two-dimensional or even a full three-dimensional model. This is however needed to represent phenomena as sea breaze along a coast, or non uniform meteorological situations, which unfortunately are not rare.

Input data for the models consist of initial profiles of wind, temperature and humidity, and the evolution of the sea temperature. These data are obtainable from standard radiosoundings and special tethered balloon soundings for the lowest level ($z \leq 500$ m)

2.2.2.- Oceanic upper layer

Although the general equations are similar, the approach of the problem has been to a slight extent different in the case of the oceanic mixed layer and thermocline. Efforts have been made to solve equations (19) to (23) using similar techniques as in the atmosphere, but much emphasis has been laid on integrated models, capable of simulating the time evolution of the upper mixed layer characteristics (depth, mean temperature and salinity). The reason for this is perhaps the lack of interest for the exact determination of current profiles, e.g., as long as their horizontal variability is not taken into account, and also the will of developing simple models of the oceanic response to atmospheric inputs which could be used as subroutines in more general problems.

Before developing these two approaches, we shall give the expression of the boundary fluxes at the top boundary and below the bottom of the mixed layer.

a) Top boundary $z = 0$

1. $\overline{u'w'_0}$, $\overline{v'w'_0}$: the determination of the stress has been detailed in part 1. A friction velocity u_* can also be introduced in the water $|\tau| = \rho_{0_w} u^2_{*_w}$ and it is related to the friction velocity in the air

$$u_{*_w} = \left[\frac{\rho_{0_a}}{\rho_{0_w}}\right]^{\frac{1}{2}} u_{*_a} \tag{35}$$

τ is directed as the wind.

2. $\rho_0 c_p \overline{w'T'_0}$ is determined from eq. (1). Usually, one considers that some part r of the solar incoming radiation is absorbed at the surface, and that the remaining part (1-r) is absorbed with depth according to a law $\sim e^{\gamma z}$. The possibility of precipitation with a temperature differing from that of the sea water is also introduced. Thus

$$\rho_0 c_p \overline{w'T'_0} = H + E + I^{\uparrow} - I^{\downarrow} - I_0 r - \rho_0 c_p P(T_p - T_s) \tag{36}$$

P is the precipitation rate, T_p the precipitation temperature and T_s the sea surface temperature

$$R = \frac{\partial I}{\partial z} = (1-r) \gamma e^{\gamma z} I_0 \qquad (I_0 \text{ positive } \downarrow) \tag{37}$$

3. $$\overline{w'S'_0} = S_0 P - \frac{E}{\rho_0 L} \tag{38}$$

where P is the precipitation rate, $\frac{E}{\rho_0 L}$ the evaporation rate and S_0 the salinity of the surface.

4. The buoyancy flux can be obtained from 2 and 3 :

$$\overline{w'b'_0} = \frac{\alpha g}{\rho_0 c_p} \left[H + E + I^{\uparrow} - I^{\downarrow} - I_0 r - \rho_0 c P (T_p - T_s) \right] - \beta g \left(P - \frac{E}{\rho_0 L}\right) S_0 \qquad (39)$$

5. $$\overline{\frac{1}{2} w' (u'^2 + v'^2 + w'^2) + \frac{p'w'}{\rho_0}} \quad . \qquad (40)$$

This is the flux of the turbulent velocity and pressure fluctuations. Near the surface, it must be equal to the rate of working by the wind, and is usually parameterized as $- c_1 u_*^3$ where c_1 is a proportionality factor.

b) Bottom boundary.

Below the mixed layer, the turbulence and turbulent fluxes vanish. If we integrate the equations, this process leads to special boundary conditions detailed below.

2.2.2.1.- Non integrated models

Mixing length hypotheses are also used in the ocean. The main problem is to find a good parameterization for the eddy diffusivities. In Mellor and Durbin's model (1975)

$$- (\overline{u'w'} \ , \ \overline{v'w'}) = e^{\frac{1}{2}} \ \ell \, S_M \left(\frac{\partial u}{\partial z} \ , \ \frac{\partial v}{\partial z}\right)$$

$$- \overline{w'T'} = e^{\frac{1}{2}} \ \ell \, S_T \ \frac{\partial T}{\partial z}$$

where $S_M(R_i)$ and $R_T(R_i)$

$$\left[R_i = \frac{\frac{\partial b}{\partial z}}{\left[\frac{\partial u}{\partial z}\right]^2 + \left[\frac{\partial v}{\partial z}\right]^2} \right]$$

take the effect of static stability on the eddy coefficients into account. The $S(R_i)$'s have been estimated using higher order closure hypotheses and laboratory experimental results.

A simplified turbulent kinetic energy balance neglecting diffusion and assuming local equilibrium is also used.

$$- \overline{u'w'} \frac{\partial u}{\partial z} - \overline{v'w'} \frac{\partial v}{\partial z} + \overline{b'w'} - \varepsilon = 0$$

$$\varepsilon = c\, e^{\frac{3}{2}}\, \ell^{-1}$$

where c is a constant and ℓ is defined as

$$\frac{1}{\ell} = \frac{1}{-Kz} + \frac{1}{\ell_\infty}$$

and

$$\ell_\infty = - c' \frac{\int_{-\infty}^{0} e\, z\, dz}{\int_{-\infty}^{0} e\, dz}$$

where c' is another constant.

These models make no assumption concerning the existence of a mixed layer. They give realistic profiles with mixed layers and thermoclines, but they require much computation work and leave some important physical processes apart.

2.2.2.2.- Integrated models

If we assume that there exists a well mixed layer (temperature, salinity, and to a lesser extent, current), we obtain the so-called slab models, and equations can be integrated over the mixed layer depth h (from - h to 0) :

$$h \frac{du}{dt} = f v h - \overline{u'w'_0} + \overline{u'w'_{-h}} \tag{41}$$

$$h \frac{dv}{dt} = - f u h - \overline{v'w'_0} + \overline{v'w'_{-h}} \tag{42}$$

$$h \frac{dT}{dt} = - \overline{w'T'_0} + \overline{w'T'_{-h}} + \frac{1}{\rho_0 c} I_0 (1-r)(1 - e^{-\gamma h}) \tag{43}$$

$$h \frac{dS}{dt} = - \overline{w'S'_0} + \overline{w'S'_{-h}} \tag{44}$$

$$h \frac{de}{dt} = \text{Prod} + \int_{-h}^{0} \overline{b'w'dz} - \left[\frac{\overline{p'w'}}{\rho_0} + \frac{\overline{w'}}{2} \overline{(u'^2 + v'^2 + w'^2)} \right]_0$$

$$+ \left[\frac{\overline{p'w'}}{\rho_0} + \frac{\overline{w'}}{2} \overline{(u'^2 + v'^2 + w'^2)} \right]_{-h} - \text{Diss} \qquad (45)$$

Here, u, v, T, S and e denote the values of the variables in the mixed layer, and geostrophic current has been omitted.

These equations involve the fluxes at the bottom boundary of the mixed layer. Taking into account the fact that discontinuities are possible at the bottom of the mixed layer for temperature, salinity, current and kinetic energy, the bottom boundary fluxes can be expressed as :

$$\overline{u'w'}_{-h} + W_e u = 0 \qquad (46)$$

$$\overline{v'w'}_{-h} + W_e v = 0 \qquad (47)$$

$$\overline{w'T'}_{-h} + W_e (T - T_b) = 0 \qquad (48)$$

$$\overline{w'S'}_{-h} + W_e (S - S_b) = 0 \qquad (49)$$

$$\left[\frac{\overline{p'w'}}{\rho_0} + \frac{\overline{w'}}{2} \overline{(u'^2 + v'^2 + w'^2)} \right]_{-h} + W_e e - W_e \frac{1}{2}(u^2 + v^2) = 0 \qquad (50)$$

where the subscript $_b$ denotes the values in the layers below the mixed layer.

The value W_e appearing in the equations is the rate of entrainment of the underlying fluid into the mixed layer, and it is equal to the time derivative of the mixed layer depths when this is increasing. However, when the depth of the mixed is decreasing, it is assumed to be completely decoupled from the ocean interior and $W_e = 0$. The physical meaning of this assumption is the fact that the ocean mixed layer cannot demix. This can be collected in a formal way using the Heaviside function

$$\theta(x) = \begin{cases} 1 \\ 0 \end{cases} \quad \text{if} \quad \begin{cases} x \geq 0 \\ x \leq 0 \end{cases},$$

$$W_e = \frac{dh}{dt}\,\theta\left(\frac{dh}{dt}\right) .$$

If we introduce equations (46) to (50) into equations (41) to (45), we get after a little algebra :

$$\frac{d}{dt}(h\,u) - f\,v\,h = \frac{\tau_{0x}}{\rho_0} \tag{51}$$

$$\frac{d}{dt}(h\,v) + f\,u\,h = \frac{\tau_{0y}}{\rho_0} \tag{52}$$

$$h\,\frac{dT}{dt} = -\,W_e(T - T_b) - \overline{w'T_0'} + \frac{1}{\rho_0 c}\,I_0(1-r)(1 - e^{-\gamma h}) \tag{53}$$

$$h\,\frac{dS}{dt} = -\,W_e(S - S_b) - \overline{w'S_0'} \tag{54}$$

$$W_e\left\{\frac{h}{2}\left[\alpha g(T - T_b) - \beta g(S - S_b)\right] + e - \frac{1}{2}(u^2 + v^2)\right\} = \mathrm{Prod}_S + c_1 u_*^3$$

$$+ \frac{h}{2}\,\overline{b'w_0'} - \frac{h}{2}\,\frac{\alpha g}{\rho_0 c}\,I_0(1-r)\left[1 - \frac{2}{\gamma h} + e^{-\gamma h}\left(1 + \frac{2}{\gamma h}\right)\right] - \mathrm{Diss} \tag{55}$$

Equation (II.27) is used as a balance, i.e. $h\,\frac{de}{dt} \simeq 0$. The production of turbulent kinetic energy is zero in the layer since there is no shear (u assumed constant). Shear can act at the bottom of the layer [third term on the left hand side of (55)], and also in a thin layer close to the surface and driven mainly by wind (Prod_S). Prod_S can be parameterized by $c_2 u_*^3$ where c_2 is a proportionnality factor. Production at the surface and turbulent flux can be taken together and $u_{**}^3 = c_1 u_*^3 + c_2 u_*^3$ used as a new turbulent velocity scale.

The problem now is to find a good parameterization for the energy dissipation. As the turbulent kinetic energy has three major sources :

- Production plus flux at the surface = $u_{**}^3 = \mathrm{Prod}_{surf}$
- Production due to shear at the bottom = $W_e\,\frac{1}{2}(u^2 + v^2) = \mathrm{Prod}_{shear}$
- Production due to buoyancy flux at the surface = $\frac{h}{2}\,\overline{b'w_0'}$ (if $\overline{b'w_0'} > 0$) = Prod_{buoy} ;

the dissipation rate is written as a sum of three parts corresponding to each particular source :

$$\text{Diss} = (1 - \phi_1)\,\text{Prod}_{\text{shear}} + (1 - \phi_2)\,\text{Prod}_{\text{surf}} + (1 - \phi_3)\,\text{Prod}_{\text{buoy}} .$$

Again, e is assumed to be of the order of u_{**}^2 (or u_*^2) . This leads to

$$W_e \left\{ \frac{h}{2} [\alpha g(T - T_b) - \beta g(S - S_b)] + c\, u_{**}^2 - \phi_1 \frac{1}{2} (u^2 + v^2) \right\}$$

$$= \phi_2\, u_{**}^2 + \frac{h}{4} [(1 + \phi_3)\, \overline{b'w'_0} - \frac{1 - \phi_3}{\overline{b'w'_0}}]$$

$$- \frac{h}{2}\, \frac{\alpha\, g}{\rho_0 c}\, I_0 (1 - r) [1 - \frac{2}{\gamma h} + e^{-\gamma h} (1 + \frac{2}{\gamma h})]$$

In the first applications, ϕ_1, ϕ_2 and ϕ_3 were taken as constant. Recently, Kitaîgorodskii developed the model for particular cases of deepening and by comparison with well documented laboratory data, he got expressions of ϕ_1 and ϕ_2 as a function of the bulk Richardson number

$$Ri_{**} = \frac{h[\alpha g(T - T_b) - \beta g(S - S_b)]}{u_{**}^2} .$$

Such models can describe the time evolution of the mixed layer if they are well calibrated.

Further improvements we would like to develop are the following :

- the development of an integrated model in parallel with a non integrated one. This will shed some light on the general validity of the parameterization of the integrated model.
- the modification of these models to introduce other effects as internal waves, e.g.
- the search for stationary or quasi-stationary solutions for this problem, and their conditions of existence.

3.- Climatic problems related to air-sea interactions

3.1.- DEFINITION OF THE PROBLEM AND JUSTIFICATION

The necessity of a better understanding of climate and climatic changes has become evident nowadays. As climatic fluctuations are governed by the climatic system components, in which the oceanic system takes a prominent part, it is understandable that the European Commission for Research in the field of climatology asks the oceanographers to undertake more research dealing with climatic problems.

Among these, some play a leading part because of their serious social and economic consequences : coldest winters for a long time in England (1962-63), USSR, Turkey (1971-72) ; highest summer temperatures and following drought in USSR, Finland and Western Europe ; persistent drought in the major countries of the Third-World : Chile (1960-69), Mexico, Sahel and Cape Verde Islands (1968-73) ; catastrophic floods all over the world (even in the central Australian desert).

The W.M.O. (World Meteorological Organization) understood it well, and its efforts to solve these problems are constant, for example through international research programs as IDOE (International Decade of Ocean Exploitation), GARP (Global Atmosphere Research Program), GATE (GARP Atlantic Tropical Experiment), WAMEX (West African Monsoon Experiment).

The problem of drought in the Sahel related to the upwelling phenomenon in the Gulf of Guinea that we are investigating is well at its place in this context.

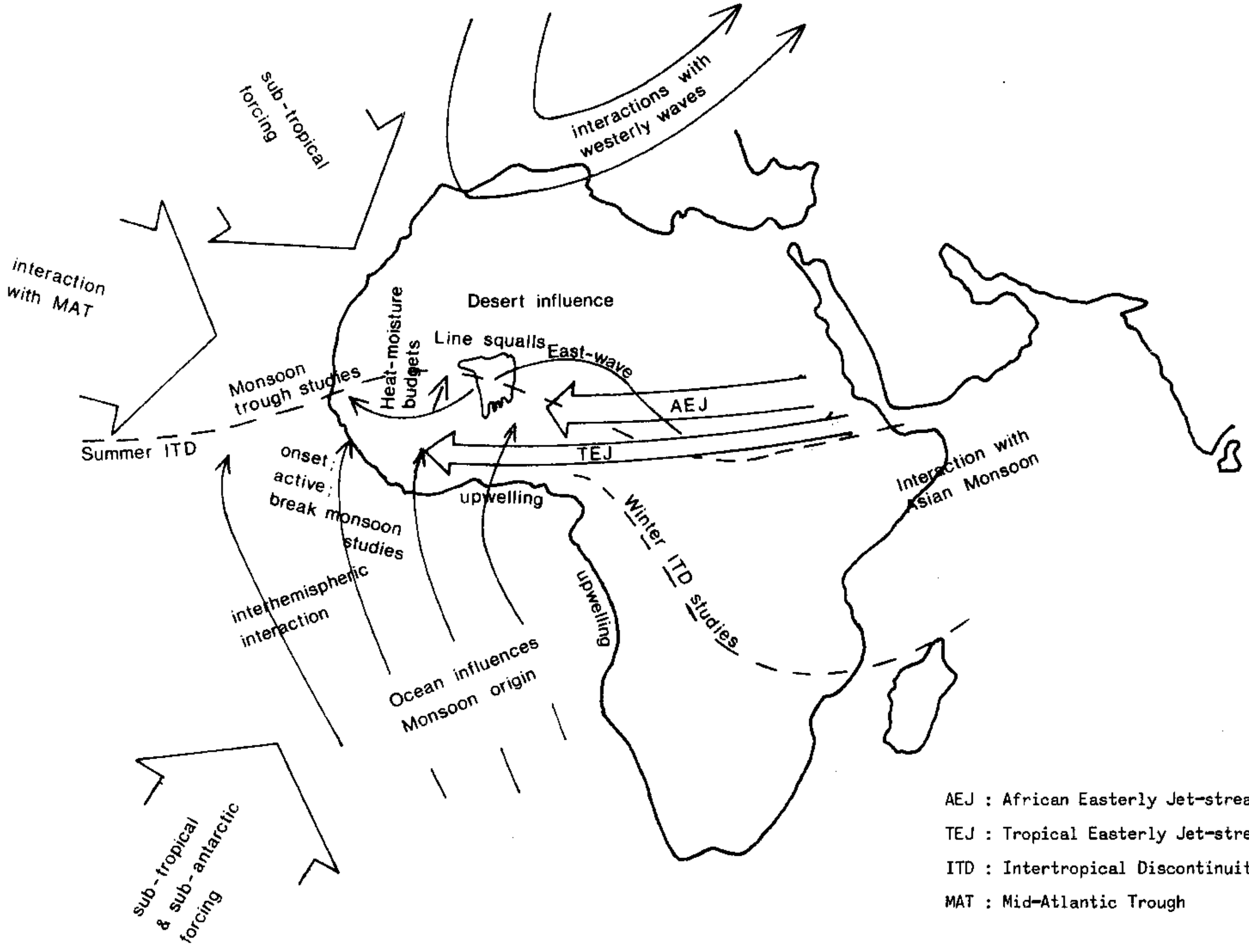
Many arguments plead in favour of this study, which are of :

1. Geographical nature :

The tropical situation of this zone first, and secondly, the proximity of the equatorial region allow theoretical studies required by meteorologists about the energetic equator fluctuations and the equatorial atmospheric and oceanic circulations (cfr. GARP), especially the ascending branch of the Hadley cell;

2. Physical nature :

- On the one hand, the existence of monsoon winds due to ocean-continent thermal gradients because of the permanent conflict between air masses under the direct influence of Açores, St. Helene and Libye anticyclones, and Saharian and equatorial troughs,



AEJ : African Easterly Jet-stream (mid-troposphere)

TEJ : Tropical Easterly Jet-stream (high troposphere)

ITD : Intertropical Discontinuity

MAT : Mid-Atlantic Trough

After WAMEX (1976).

fig. 6.