

# Changes in arable land-use choice under climate change: the case of the loam region in Belgium\*

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## Abstract

We analyze arable land-use changes under climate change in the loam region in the centre of Belgium. An agro-economic model which is calibrated based on land-use observations in the period 2009-2014 is used for this purpose. We consider 20 year-series of projected simulated yields in the different climatic scenarios. We show that adaptation to climate change through crop management has a positive impact on individual utility in the loam region. In particular, land-use adaptation consists of an increase in the share of land allocated to wheat and of a decrease of those for summer crops and barley. Finally, irrigation is not always justified in the loam region under climate change.

**Keywords:** farmer's adaptation, climate change, risk aversion, agro-economic model, positive mathematical programming.

**JEL:** C61, Q12, Q54

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# 1 Introduction

Scientific climate change reports (IPCC (2007, 2008, 2014) [15] [16] [17]) project important economic and environmental impacts on the agricultural sector. Specifically, changes in precipitation, temperature, extreme weather events and CO<sub>2</sub> are expected, which are not uniform across European regions. Indeed, such changes may positively or negatively impact the agricultural activity of the different regions. However, some challenges are common in Europe such as the increase in the percentage of surface area under water stress (cf. Iglesias and Garrote (2015) [14]). In this context, adaptation is necessary to mitigate climate change impacts in agriculture.

There is a relatively recent but large literature about adaptation strategies to climate change. Studies have considered different adaptation measures to deal with the challenges; these includes irrigation (cf. Finger et al. (2011) [6]), land-use (cf. Kaiser et al. (1993) [22]), technology adoption (cf. Foudi and Erdlenbruch (2012) [10]), financial support (cf. Berrang-Ford et al. (2011) [2]) among others. We concentrate on arable land-use changes under climate change. Different methods are employed in the literature to study this topic, e.g. econometric models (see Seo and Mendelsohn (2008) [26], Mu et al. (2013) [23]) and integrated models (see Kaiser et al. (1993) [22]) at different levels of decision-making (global, regional and farm levels). Kalaugher et al. [22] call for the use of integrated models with interdisciplinary approaches. They argue that much of the recent literature deals with the assessment of climate impacts in an inadequate way because of the "profound failure of knowledge" in the different disciplines.

In this work, we focus on the use of an agro-economic model at the farm level. We use a Positive Mathematical Programming (PMP) framework which takes into account risk in the decision-making. Some authors have already used this methodology (e.g. Paris and Arfini (2000) [24], Cortigiani and Severini (2012) [3], Petsakos and Rozakis (2015) [25]). The main differences between these previous works lie in the type of utility function considered and estimated parameters. Paris and Arfini (2000) use a mean-variance approach with CARA (constant absolute risk aversion) preferences and the 3 classical steps of the PMP model (cf. Howith (1995) [13], Heckelei (2002) [11]), namely the addition of calibration constraints to a

linear programming (LP) model, the estimation of dual values of the constraints to derive the non-linear terms and the introduction of these non-linearities in the LP calibrated model. However, the risk aversion coefficient is considered as an exogenous parameter in the model. This coefficient is estimated by Cortigiani and Severini (2012) along with the non-linear cost function and the resource shadow price with a maximum entropy program (cf. Heckeley and Wolff (2003) [12]). In contrast to the previous works, Petsakos and Rozakis (2015) have recently argued that the two main assumptions on which linear E-V models are based, namely the normal distribution of wealth and the exponential utility function, cannot be easily accepted and calibration models based on finding a true value of the risk aversion coefficient and/or a quadratic cost function cannot be easily implemented. Thus, they propose a DARA (decreased absolute risk aversion) utility function and a three stage PMP approach in which the variance matrix is rectified.

We adapt the Cortigiani and Severini (2012) approach to simulate land allocation of a representative farm. In that way, estimation of the risk parameters is more credible since the "true" value of the risk aversion coefficient is estimated from past production experiences of a representative farmer. Moreover, the implementation of the PMP model is simplified and prices and yields are considered as independent random variables.

Most of the European empirical studies about climate adaptation strategies are made in Southern Europe where changes in weather conditions due to climate change would be more pronounced (cf. [7]). Studies in Atlantic and continental regions are limited instead are also expected such as the increase in extreme events and shifts in land-use (cf. Iglesias and Garrote (2015) [14]). We applied the theoretical model to a typical arable farm of the loam region in Belgium. The loam Region located in Central Belgium and formed on quaternary loess, has the best soils for arable agriculture. Fairly large farms grow mainly cereals, maize, sugar beet and potatoes. The aim of the paper is to analyze optimal land-use changes with and without weather related stress for the current years and under climate change.

This paper is organized as follows. In section 2, we present the agro-economic model. We describe data of the loam region and assumptions made for the modeling in section 3. Results for the numerical example are described in section 4. We finally conclude and

provide some possible extensions to this work in section 5.

## 2 An agro-economic model of land-use choice

### 2.1 Agronomic Component

The agronomic component of the model quantifies the technical productivity. A coupled biomass and water balance model (cf. Gobin (2010, 2012) [8], [9]) was run under current and future climatic conditions according to IPCC compliant scenarios. The water balance consists of an atmospheric compartment with rainfall, temperature and evapotranspiration as input variables; a soil compartment based on soil-water physical properties and equations; and, a coupled soil moisture-biomass model which is optimised during subsequent model runs. The parametrisation of the different equations is crop specific. The water balance is operated as a single-layer bucket model confined between field capacity and permanent wilting point with a variable crop rooting depth during the growing season. The soil available water capacity for a loam soil is assumed 200 mm/m. The fraction of water available to the plant depends on the drought sensitivity of the crop type, with a default value of 0.5 for arable crops. We compare four different impact scenarios: (1) current normal farmers' yields; (2) current yields without weather-related stress; (3) yields under climate change; and, (4) yields under climate change without climate-related stress. We assume that climate-related stress can be alleviated with adaptation and mitigation measures.

### 2.2 Economic Component

The economic component modelling is based on a Positive Mathematical Programming (PMP) approach. More specifically, we adapt the PMP model proposed by Cortigiani and Severini (2012) [3] for a representative farm.

The optimization problem consists of finding the vector of shares of land allocated to the different crops,  $x$ , that maximizes the expected utility,  $E(Z)$ , constraint by equation (2) for available land:

$$\max_x E(Z) = E(g)^\top x - d^\top x - \frac{1}{2}x^\top ex - c_i^\top x - \frac{1}{2}\phi x^\top \Sigma_g x \quad (1)$$

$$a^\top x \leq S \quad [\lambda] \quad (2)$$

where  $E(g)$  is the vector of expected unitary gross margin,  $d$  and  $e$  are parameters of the quadratic cost function,  $\phi$  is the farm specific coefficient of risk aversion and  $\Sigma_g$  is the covariance matrix of unitary gross margins.  $c_i$ , the (per hectare) irrigation cost,  $a$ , the unitary vector and  $S$ , the total amount of land available, are exogenous parameters.

$E(g)$  and  $\Sigma_g$  are calculated from data and outputs of the agronomic model over a period of  $T = 20$  years. Specifically, prices and yields are treated as independent random vectors which follow a discrete uniform distribution in the interval  $[0, T]$ . Thus,

$$E(g_k) = E(p_k) * E(y_k) - c_k^s \quad \forall k \in K, \quad (3)$$

where

$$E(p_k) = \frac{1}{T} \sum_t p_k^t, \quad (4)$$

$$E(y_k) = \frac{1}{T} \sum_t y_k^t. \quad (5)$$

and  $c_k^s$  is the structural cost (per hectare) of the crop  $k$ .

The variance-covariance matrix  $\Sigma_g$  is defined by the classical equations:

$$\Sigma_{g(k,k)} = \frac{1}{T-1} \sum_t (g_k^t - g_k)^2, \quad (6)$$

$$\Sigma_{g(k,l)} = \frac{1}{T-1} \sum_t (g_k^t - g_k)(g_l^t - g_l) \quad (7)$$

$$k, l \in K, \quad k \neq l \quad (8)$$

where  $g_k^t = p_k^t y_k^t - c_k^s$  is the gross margin value from production of the  $k^{th}$  crop at year  $t$  and  $g_k$  is the mean of the gross margins distribution. Irrigations costs,  $c_i$ , include

fixed and variable costs, which are derived from data of the study area and agronomic simulations (see details in section 3.3). Parameters  $d$ ,  $e$ ,  $\phi$  and the vector of dual values,  $\lambda$  are estimated with a maximum entropy program (see appendix A.1).

### 3 Data of the loam region (Belgium)

We apply the theoretical model to a representative and typical arable farm of the loam region. Situated in the centre of Belgium, the loam region is the most productive agricultural region in Belgium because of the characteristics of the soil (cf. [1]). Typical large arable farms are representative for the study area and are specialized in the cultivation of cereal grains, potatoes and sugar-beets. In what follows, we detail the real and simulated data that are used in the optimization model.

#### 3.1 Simulated agronomic data

From the agronomic model, we obtain series of 20-years simulated average yields for 5 crops (Wheat, Barley, Potato, Sugar beet and Grain Maize) and 4 scenarios: current normal yields and projected yields under climate change, with and without weather related stress. Results are summarized in figure 1. We can observe that, under climate change, higher winter cereal yields and lower summer crops yields are projected with respect to the current normal scenario.

#### 3.2 Economic data

Yield prices of the 5 crops over a period of  $T = 20$  years correspond to the period 1993-2012 (cf. ADSEI index [27] and cf. [28] for sugar beet prices), for which projected yields are simulated in the baseline scenario (current normal yields).

Typical structural costs (per hectare) for each crop in the loam region are taken from the Flemish official reports [6], [20], [21] and are summarized in table 1. From equations (3), (6) and (7), we subsequently calculated the expected revenue  $E(g)$  and the covariance matrix  $\Sigma_g$  which are inputs of the economic model.

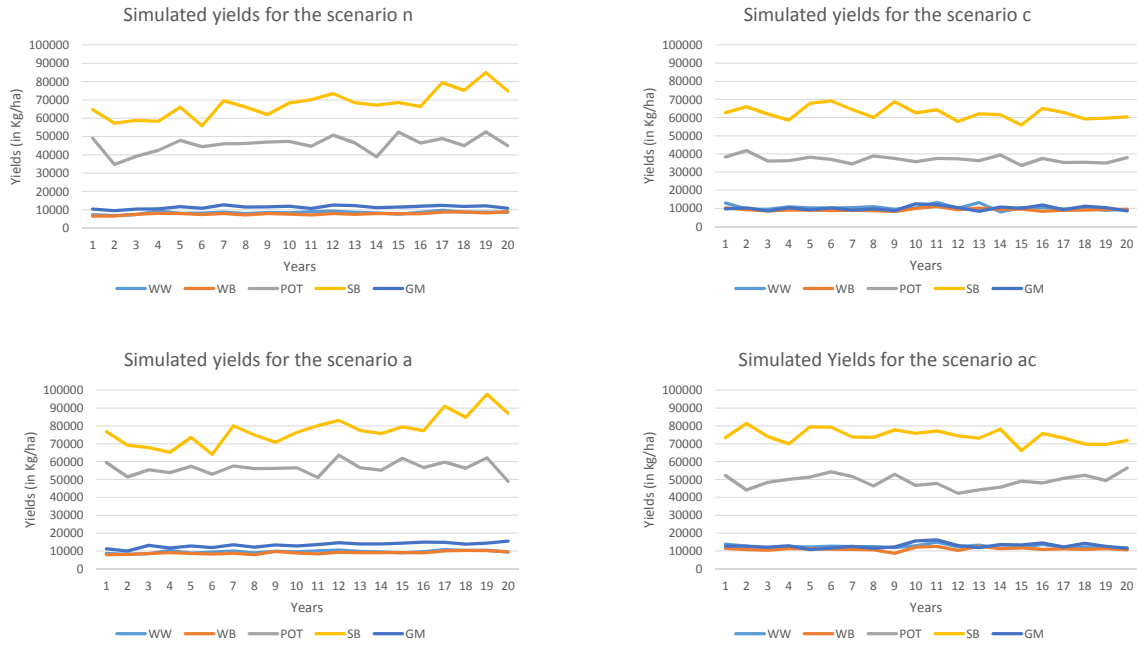


Figure 1: Simulated average yields for Winter Wheat (WW), Winter Barley (WB), Potato (POT), Sugar beet (SB) and Grain Maize (GM) for the different climatic scenarios: Current normal years with weather related stress (n) and without stress (a), future climate change with stress (c) and without stress (ac).

Irrigation costs are taken into account in the scenarios "without weather related stress": as fixed and variable costs. Fixed costs include investments in materials and technology for irrigation and are estimated on average around 225 €/ha in Belgium (Janssens 2015 [19]). Variable costs are obtained by multiplying the quantities of water dedicated to each crop (in mm), which are outputs of the agronomic model, and the price of water, that is around 50 € per water turn of 25 mm and per hectare (Janssens 2015 [19]).

Parameters (in €/ha)	$c_1^s$	$c_2^s$	$c_3^s$	$c_4^s$	$c_5^s$
Values	606	520	1 267	798	583

Table 1: Structural costs in the loam region for crop 1: Winter Wheat, 2: Winter Barley, 3: Potatoes, 4: Sugarbeet, and 5: Grain Maize.

### 3.3 Estimated values of Operating Costs, shadow price and risk aversion coefficient

The output values of the calibration program (see Appendix A.1) provide the inputs of the optimization model. Indeed, parameters of quadratic and "risk" costs,  $d$ ,  $e$ ,  $\phi$  and the shadow price value of the resource constraint,  $\lambda$  are estimated with the calibration program by taking arable land-use observations of typical farms of the loam region (cf. [27]) for the period 2009-2014. As in [12], for the current case of one resource constraint, the elements of vector  $d$  are not identified and therefore set to zero. Calibrated values are listed in Table 2.

Parameters	Unit	Values
$e_1$	€/ha.ha	0.98
$e_2$	€/ha.ha	0
$e_3$	€/ha.ha	86.468
$e_4$	€/ha.ha	77.6
$e_5$	€/ha.ha	102.289
$\phi$	<i>unitless</i>	1.010351E-5
$\lambda$	€/ha	269

Table 2: Calibrated values/outputs of the maximum entropy model.

## 4 Results for the loam region

In this section, optimal results for the different scenarios are described: current normal yields with (n) and without weather related stress (a) and projected yields under climate



change with (c) and without weather related stress (ac). In case of zero weather related stress, we assume that farmers apply adaptation measures such as water conservation, irrigation and drainage, mulching with crop residues.

#### 4.1 Land-use adaptation to climate change

The optimal surface ( $x_k$ ,  $k = 1..5$ ) allocated to winter wheat, winter barley, sugar beet, late potatoes and grain maize for a farm-type of 100 ha, and the individual expected utility ( $E(U)$ ) obtained for the different scenarios are presented in Table 3.

First of all, we analyze land-use adaptation to climate change with weather related stress, i.e. the comparison between the "n" and "c" scenarios in Table 3. Simulations results show that the share of land of wheat increases by 24 hectares (ha), while the surface allocated to barley, sugar beet, potatoes and grain maize decreases by around 10, 6, 5 and respectively 3 ha. More specifically, winter barley gradually phases out and grain maize covers a small acreage in projected simulations under climate change. These results are in the same line as agronomic simulations for projected yields (cf. section 3.1) except in the case of winter barley where yields are expected to increase while the share of land is declining. This is related to the fact that the calibrated value of the non-linear parameters associated with winter barley are zero and the "risk cost" (which is defined in the last term of equation (1)) of winter barley (of around 69 €/ha) is greater than that the winter wheat (of around 61 €/ha). Thus, among winter crops, it's preferable to keep the crop with the lowest risk, i.e. winter wheat. Furthermore, climate change adaptation leads to an increase of individual utility of around 6 000 euros, which corresponds to a significant increase of 9 points with respect to the current normal years.

However, results change if we assume that crops grow without weather related stress in the current normal and climate change scenarios (i.e. a and ac scenarios in Table 3). Tendencies concerning land-use adaptation are maintained, but the expected utility now decreases by around 9 069 euros (which corresponds to the 14 %). Indeed, an assumption of an increase in irrigation needs is considered under climate change (cf. [18]), leading to higher irrigation costs and an important loss on the farmer's individual utility.

	unit	Scenarios			
		n	c	a	ac
Winter Wheat	ha	52.097	76.174	57.048	78.336
Winter Barley	ha	9.74	0	0.443	0
Late Potatoes	ha	15.152	8.957	17.597	8.392
Sugar beet	ha	18.622	13.935	21.877	13.272
Grain Maize	ha	4.389	0.934	3.036	0
Expected utility	€	63 064	68 983	64 839	55 770

Table 3: Surfaces allocated to the different crops and expected utility for the different scenarios: current year with (n) and without weather related stress (a) and climate change with (c) and without weather related stress (ac) scenarios

## 4.2 Different price scenarios

Following climate change projections, prices should be one of the more sensitive parameters affected by climate change. In this section, we simulated a change in crop prices under climate change scenarios and analyzed land-use adaptation under these assumptions. We considered different simulated projections of crop prices consistent with existing literature (see [5] for a review). Simulated increases were as high as 150% for all crop prices (scenario "all crops"), in cereal prices only (scenario "cereals only"), and in all crop prices except potatoes (scenario "all except potatoes"). The latter scenario is particularly interesting for the Belgium case. Indeed, the demand for potatoes in Belgium is high and stable because of the processing industry.

Results of prices scenarios in Table 4 show that summer (resp. winter) crops used a higher (resp. lower) share of land under the assumption of a general increase of crop prices under climate change scenarios. The adaptation behavior under higher crop prices lead to land-use results closer to the baseline case (current normal year scenario). However, if we consider just higher cereal prices and stable prices for sugar beets and potatoes because of current stable production and nearness to the processing industry in the study area, preference was given to the cereals and summer crops gradually disappeared in the

landscape. Logically, since we assumed that farmers behaved optimally, the individual expected utility increased with higher prices.

Percentage of changes	Prices Scenarios		
	↗ all crops	↗ cereals	↗ all crops except Potatoes
$x_1$	~ 0 to -34%	~ 0 to +24%	~ 0 to -18%
$x_2$	0	0	0
$x_3$	~ 0 to +36%	~ 0 to -100%	~ 0 to -100%
$x_4$	~ 0 to +155%	~ 0 to -94%	~ 0 to +156 %
$x_5$	~ 0 to +125%	~ 0 to +416%	~ 0 to +102%
$E(U)$	~ 0 to +330%	~ 0 to +201%	~ 0 to +301%

Table 4: Changes (in %) of land-use and expected utility for different prices scenarios under climate change.

## 5 Conclusion and Extensions

In this paper, we analyzed land-use adaptation to climate change in arable farms of the loam region in Belgium. We built an interdisciplinary model with agronomic and economic components. From the agronomic model, we obtained 20 years-series of projected yields for the most representative crops in the study area (cereals, maize, sugar beet and potato) and for different scenarios: current normal farmers' yields (the baseline case); current yields without weather-related stress; yields under climate change; and yields under climate change without climate-related stress. These simulated yields were used ~~such~~ as inputs to the economic model, which is divided in two steps: firstly, a maximum entropy program where the shadow price of the land, the non-linear cost parameters and the risk aversion coefficient were calibrated based on land-use observations in the study area; secondly, a main optimization program where calibrated parameters were introduced to explain optimal land-use changes under climate change.

The main contribution of the paper is assessing the impact of climate change in optimal land-use choices in the loam region. Situated in the centre of the country, the loam region is characterized by fertile soils. We showed that land-use adaptation to climate change consisted of an increase in the share of land allocated to wheat and a decrease in the share of land allocated to barley, sugar beet, potatoes and grain maize, with and without weather related stress. We concluded that adaptation to climate change through crop management has positive impacts on the farmer's individual utility, leading to a gain of around the 9 % with respect to the baseline scenario. Nevertheless, irrigation is not justified in the loam region under climate change because of the highest irrigation costs in Belgium. Moreover, by simulating a likely increase in crop prices under climate change, optimal land-use choices approximated progressively the current farmer's behavior and highest revenues should be expected in case of adaptation.

Several extensions of this work are possible: first, we could improve the calibration program by adding the main rotation constraints in the study area, avoiding then the disappearance of winter barley under climate change; secondly, other types of farmer's adaptation strategies could be analysed; thirdly, policy recommendation could be proposed

for the particular case of the loam region; finally, a sensitivity analysis with different projected prices in the future could be made in the paper.

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## A Appendix

### A.1 The maximum entropy program

We use a maximum entropy program which was introduced by Heckelei and Wolff. H (2003) [12] and adapted by Cortignani and Severini (2012) [3] with the aim to consider risk aversion.  $K$ , the number of crops,  $Q$ , the number of support points and  $N$ , the number of observation years, are the sets of the model.

The objective of the program is to find the vectors of probabilities  $w_{kq}^n$ , parameters of the quadratic costs,  $d_k$ ,  $e_k$ , the absolute risk aversion coefficient,  $\phi$ , and the shadow price,  $\lambda$  that maximize the entropy level,  $H$ , described in equation (9), subject to first order conditions of the problem described in section 2.2 (equation (10)) which are evaluated in observations years, to error terms (11) and land (12) constraints and vector of probability condition (13). Error terms (11) are reparameterised as expected values of a discrete probability distribution and can be represented as the multiplication of  $z_q$ , the support values with the vector of probabilities  $w_{kq}^n$ .

$$\max_{w_{kq}^n, d_k, e_k, \phi, \lambda} H = - \sum_{n, k, q} w_{kq}^n \ln w_{kq}^n \quad (9)$$

s.t.

$$E(g_k) - d_k - e_k(x_k^n - \epsilon_k^n) - \sum_{g_k} \phi(x_k^n - \epsilon_k^n) - \lambda^n = 0 \quad (10)$$

$$\epsilon_k = \sum_q z_q w_{kq}^n \quad (11)$$

$$\sum_k x_k^n - \epsilon_k^n = S \quad (12)$$

$$\sum_q w_{kq}^n = 1 \quad (13)$$

$$\forall k \in K, \quad \forall q \in Q, \quad \forall n \in N.$$