OPTIMAL INVESTMENT STRATEGY UNDER UNCERTAINTY

IN THE BELGIAN ENERGY SYSTEM

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I. Introduction

OBJECTIVE OF THE PAPER

Investment decisions for the Belgian energy system, as any long-term investment decisions, have a strong component of uncertainty. Because the life time of the technologies covers many years, one has to take account of things that may happen in the future and this brings an element of uncertainty.

The objective of this paper is to compare the solution of the stochastic strategy with solutions of deterministic strategies, when there is uncertainty about the CO2 emissions that will be imposed after Kyoto[1]. The respective CO2-emissions paths and their costs will be compared, as well as the primary energy input, the final energy demand and the choice of technologies.

STRUCTURE OF THE PAPER

The first chapter gives the theoretical background behind the practical application for the Belgian energy system. First the concepts of risk and uncertainty are introduced and it is shown how risk can be represented by probability distribution functions. Then the approach proposed in the economic theory for decision-making under uncertainty, which is based on maximisation of the expected utility, is briefly explained. Finally, this approach is compared with some other approaches proposed for decision making under uncertainty.

The second chapter describes how the problem of decision making under uncertainty was implemented in "stochastic Markal", the linear-programming model used in this study

In the third chapter, we present the results of "an optimisation of investments for the Belgian Energy system under uncertainty", generated by the stochastic Markal model. In section A we give the model assumptions. In the sections B to E, we give the results of the stochastic strategy and compare them with results from deterministic strategies. In section F, we make some sensitivity studies regarding assumptions adopted in the model, e.g. we look at the impact of putting an end to the investments in nuclear power plants.

[1] Other elements of uncertainty, e.g. the availability and characteristics of some future technologies or uncertainty about future energy-prices are only touched but not studied in detail in this document.

II. The theory of decision making under uncertainty

A. Uncertainty and risk

1. Definitions of risk and uncertainty

People are certain if there is only one possible outcome. If there is more than one possible outcome, one deals with risk or uncertainty.

Knight (1921) proposed to speak of *risk* if the possible outcomes and their probabilities are well known, in other words if the probabilities are an objective fact and of *uncertainty* if the probabilities are not well known.

In practice, when the objective probabilities of the outcomes[2] are not known, *estimates* are used, and in that case one speaks of *subjective probability.* To obtain estimates of probabilities Raiffa e.g. (1970) has proposed a structure for a dialogue between experts. With these subjective probabilities, the choice under uncertainty can be treated in the same way as decision making under risk. One additional step is however useful: a sensitivity analysis around the estimated probabilities to check the robustness of the solution.

2. Risk represented by probability distribution functions

The nature of the risk depends on the relative position of the possible outcomes and on their (subjective) probability. For instance, one can feel intuitively that the risk is smaller if the possible outcomes are close to each other or if unwanted outlying outcomes have smaller probabilities. So, in order to describe the nature of the risk, one will describe the relative positions and the probabilities of the outcomes. These can be described either through the *probability density function* or through the *cumulative distribution function*. One widely used distribution is the *normal distribution* which is completely characterised by its mean and variance.

B. Decision making under risk: the economic theory approach

1. The utility function and the concept of risk aversion

"In Victorian days 'utility' was thought of as a numeric measure of a person's happiness. ... It was natural to think of consumers making choices so as to maximise their utility, that is to make themselves as happy as possible. ... The theory of consumer behaviour has been reformulated in terms of consumer preferences, and utility is seen only as a way to describe preferences" (Varian, 1993).

Mostly a utility function, u(..), of an individual has the following characteristics:
1) the utility increases if the wealth, x, increases: $du / dx > 0$
2) the marginal utility decreases if the wealth increases: $d^2u / (dx)^2 < 0$.

[2] If different "outcomes" are possible, we will also refer to this as "states of nature" or "states of the world".

If the second characteristic is true, i.e. if the utility function is concave, then at any level of wealth, x, the utility gain from an extra dollar is smaller than (the absolute value of) the utility loss of having a dollar less.

Most individuals have an increasing and concave utility function; this means that their utility increases if their wealth increases but the marginal utility decreases if the wealth increases. Risk aversion finds its origin in this decreasing marginal utility. In annex, the concept of risk aversion and its measuring is further examined.

2. Maximising the expected utility

In the economic theory, the main approach for decision making under risk is maximising the expected utility. To calculate the expected utility, we take the sum of the utility under all possible outcomes weighted by the probability of the outcomes. So, on one hand it is necessary to know the probability distribution of the outcomes, on the other hand it is necessary to know the utility function of the decision maker.

A risk, represented by a cumulative distribution function, F(x), is then evaluated by an individual with utility function U(...), by calculating the expected utility:

$$U(F) = \int u(x)\, dF(x).$$

If such an individual has to choose between different risks, he will choose the one with the highest expected utility, for him (which depends on his utility function).

C. Decision strategies under uncertainty in practice

If in theory the main approach for analysing optimal decision under risk is based on the maximisation of the expected utility, in practice this approach is not always followed. We will briefly described different approaches used and try to compare the solutions they generate. The approaches considered are: deterministic analysis, stochastic strategy and minimax strategy.

1. Deterministic strategies

In the deterministic analysis, one calculates the best actions to be taken for an outcome, under the assumption that it is for sure that this outcome will take place. Because in reality decision makers are not 'clairvoyant' and therefore do not know which outcome really will take place, they calculate the best solutions for a range of possible outcomes. Thus, a deterministic strategy leads to as much different advices as there are different scenarios. Therefore this strategy gives only an indication of the range of actions. It can be used in a first step, but is not suited to give a final strategy to apply under uncertainty.

If a deterministic strategy is calculated for a specific outcome, it is possible that this outcome will be the true one. But it is also possible that it is not the true one. In that case the cost that will be incurred is higher than anticipated. The expected cost of a deterministic strategy can be much larger than the expected cost of the stochastic strategy (Amit Kanudia and Richard Loulou, 1997, Birge J.r. and Rosa C.H., 1996).

2. Stochastic Strategies

Ideally the modelling framework described in the previous section should be applied : maximising the expected utility. However it is rather difficult to find a specific utility function and distribution function which yields a tractable form for the expected utility, when the situation considered becomes more complex. Therefore in practice when considering investment decision under uncertainty, the mean-variance model is applied. Other approaches have tended to linearise the objective function.

a) The mean-variance model

The mean-variance model maximises the objective function :

$$\mu - \lambda * \sigma^2 ,$$

where μ is the expected return and σ^2 is the variance of the disposable income. The variance is used as an indicator of the risk of the return and λ is a parameter reflecting the degree of risk aversion of the decision-maker.

This model corresponds only exactly to the expected utility approach under very stringent conditions on the utility function or on the probability distribution function :

1. the form of the utility function is quadratic, because the derivatives of the third order and higher are zero for such functions and therefore the objective function can be written in function of only the expected wealth and the variance of the expected wealth. However the quadratic utility function has the unrealistic properties of satiation and increasing risk aversion.
2. the end of period wealth is normally distributed, because then the probability distribution is entirely characterised by the mean and the variance

But as Varian mentions “Even for non-normal distributions, which cannot be completely characterised by their mean and variance, the Mean-Variance model may well serve as a reasonable approximation to the expected utility model” (1993).

This can be shown by expanding an individual’s utility as a Taylor serial around its expected end of period wealth (Chapter 3 of Huang and Litzenbergers “Foundations for financial economics” (1988)):

When u(..) is the utility function, E{..} is the expected value and
X is the uncertain end of period wealth, we have:

(i) $$u(X) = u(E\{X\}) + u'(E\{X\})(X - E\{X\}) + \tfrac{1}{2}u''(E\{X\})(X - E\{X\})^2 + R_3$$

with $R_3 = \sum_{n=3}^{\infty} 1/n!\ u^{(n)}(E\{X\})(X - E\{X\})^n$; $u^{(n)}$ is the n-th derivative of u.

Assuming that the Taylor series converges and that the expectation and summation operations are interchangeable, the individual’s expected utility may be expressed as:

(ii) $$E\{u(X)\} = u(E\{X\}) + 1/2!\ u''(E\{X\})\ \sigma^2(X) + E(R_3),$$

with $R_3 = \sum_{n=3}^{\infty} 1/n!\ u^{(n)}\ (E\{X\})\ m^n(X)$; $m^n(X)$ denotes the n-th central moment of X.

The first two terms of the equation under (ii), indicates that the individual has *a preference for expected wealth and an aversion to variance of wealth*, which is consistent with the usual assumption regarding utility functions. These properties are also completely captured in the Mean-Variance model.

The remainder term $E(R_3)$ in the equation under (ii), contains central moments of orders higher than the second. Therefore, the expected utility cannot be defined in general cases by using only the mean and the variance of the wealth distribution. However, a model that uses only the mean and the variance to choose between portfolio, can give a solution that approximates the solution maximising the expected utility, assuming that the last term is close to zero.

When λ is assumed to be zero, the decision-maker is assumed to be risk-neutral. Besides the assumption regarding the attitude towards risk of the decision maker, this assumption has also a practical advantage : the objective function becomes linear and linear optimisation programs are much more powerful than non linear ones.

In some applications the variance/standard deviation is replaced with the upside standard deviation. Then one assumes that the investor is not concerned about downward deviations of the costs or that for wealth levels larger than the expected wealth he is risk neutral. As such this is difficult to justify, but it is mostly taken as an approximation for the impact of the 2^{nd} and higher order moments in the Taylor expansion.

b) Other ways of representing risk aversion

Risk-averse decision makers have a concave utility function ($u'(x)>0$, $u''(x)<0$), therefore it can not be represented by linear functions. In order to allow for a solvable objective function one may consider using a linear approximation of this utility function, as has been done for MARKAL-ED. Also an approach which is proposed for investment decisions in firms is, instead of a positive λ, to add a constraint putting an upper bound on the loss.

3. The minimax regret method

The minimax regret method has also been applied for some problems of decision making under uncertainty. The regret of a strategy s under outcome z, R(z,s), is defined as the difference between the cost incurred when strategy s is used and outcome z occurs, C(z,s), and the minimum cost that can be incurred under outcome z by any possible strategy[3] t:

$$R(z,s) = C(z,s) - \min_{t \in S} C(z,t);$$

[3] The minimum cost that can be incurred for an outcome, is the cost under the deterministic strategy that assumed from the start that this outcome would realise for certain and had the luck to be correct.

The maximum regret that can be incurred under a strategy s, occurs for the outcome, z, for which the difference between the cost under this strategy, s, and the minimum cost for any possible strategy is the largest: $\underset{z \in Z}{Max}\left[C(z,s) - \underset{t \in S}{Min} C(z,t) \right]$.

The minimax regret method will then select a strategy, s*, with the smallest (=>min) maximum regret that can be incurred. In mathematical notation this is given by:

$$s^* \in \text{ArgMin (Max R(z,s))}$$

or

$$s^* \in \textbf{Arg} \underset{s \in S}{Min} \left\langle \underset{z \in Z}{Max} \left[c(z,s) - \underset{t \in S}{Min} C(z,t) \right] \right\rangle$$

with s ∈ {all possible strategies} and with z ∈ {all possible outcomes}

We want to stress that this strategy minimises the maximum *regret* and not the maximum *cost*.

An advantage of the minimax regret criteria is that it only needs a list of possible outcomes and no probabilities of the outcomes. Loulou and Kanudia have experimentally verified that the solution of this minimax regret criterion only depends on the two extreme scenarios. They also made a comparison between deterministic, stochastic and minimax strategy for the optimisation of investment strategies under uncertainty for the energy system of the province of Québec (Canada). In their comparison, five possible limits on the maximum amount of CO2 emissions during the period 1990-2030 are considered:

1. on average emissions have to stay at the 1990 level;
2. on average emissions have to be below the level of 1990 with 10%,
3. on average emissions have to be below the level of 1990 with 20%,
4. on average emissions have to be below the level of 1990 with 30%
5. on average emissions have to be below the level of 1990 with 40%.

Until 2012 it is unknown which of the five limits will have to be satisfied.

The stochastic strategy assumes until 2012 that each of the 5 outcomes has a probability of 0.20 to take place. The minimax regret strategy considers until 2012 that each of the 5 outcomes can take place. After 2012 the best strategy is followed taking in account the true outcome and the past actions. The authors calculated the total discounted costs for the whole period for each of the strategies. In the table below, each row presents data concerning the solution of one of the 7 strategies. In the column with heading “0%” the regret is given if in 2012 it turns out that the yearly emissions have to be 0% lower than in 1990, due to the cumulative constraint. We have seen already that this regret is the difference between the cost under the outcome and the lowest possible cost that can be incurred for a strategy under this outcome[4].

[4] We can also see this regret as the difference between a strategy and the deterministic strategy that was luckily correct, since no strategy can be less costly.

Table 1: Comparison of strategies for decision making under uncertainty by comparison of the expected cost and the possible regrets.

	0%	10%	20%	30%	40%	Expected Cost (E.C.)	E.C. relative to E.C. stochast.	Max. Regret	diff. in max regret with MMR	diff in max regret relative to E.C stochast
Det 0%	0	639	2714	7416	113059	1283900	1,01812	113059	109748	0,08703
Det 10%	314	0	784	2845	62559	1272435	1,00903	62559	59248	0,04698
Det 20%	1377	408	0	532	17868	1263172	1,00168	17868	14557	0,01154
Det 30%	3302	1835	584	0	4635	1261206	1,00013	4635	1324	0,00105
Det 40%	9183	7343	4702	2025	0	1263785	1,00217	9183	5872	0,00466
stochast	3526	2118	811	88	3023	1261048	1,00000	3526	215	0,00017
MMR	3311	2010	837	213	3308	1261070	1,00002	3311	0	0,00000

From the data in Kanudia and Loulou, it is evident that none of the deterministic strategies performs nearly as well as the stochastic or the minimax regret strategy, the expected cost of their solutions is much larger, as well as the maximum regret that can be incurred. For this example the solution of the stochastic and the minimax regret strategy are very close to each other. The expected cost of the minimax regret strategy is only 0,002% larger than the expected cost of the stochastic strategy, while the maximum regret of the stochastic strategy is 0,017% larger than the expected cost of the stochastic strategy.

4. Conclusion

The deterministic strategies are the least suited to give policy advise under uncertainty. First, the expected cost of the individual deterministic strategies exceeds the expected cost of the other strategies (stochastic, minimax regret). A second reason is that the other strategies give 1 policy advise, while the deterministic strategies give a different solution for each possible outcome. And it was found empirically that for complex problems, the solution from the stochastic strategy can not be reconstructed by weighing the solutions of the deterministic strategies. Also, if one weighs the solutions for the different possible deterministic outcomes in order to get 1 policy advise, one is more vulnerable to subjectivity than when one tries to estimate the probabilities of the different possible outcomes in the beginning. Including the estimates of the probabilities in the problem in the beginning, means using the extra information when generating an optimal solution and this extra information will improve the solutions. As Raiffa (1970) argues, it is better to include subjective information than not including the information at all.

The minimax regret criterion does not need any information on probabilities. However in fact it looks only at the extreme outcomes whereas the stochastic strategy takes account of the whole probability distribution. Therefore if there is especially uncertainty about the extreme outcomes and no or little uncertainty about the middle part of the distribution this advocates for using the stochastic criterion rather than the minimax regret criterion, whereas if the extreme outcomes are more certain and the rest of the distribution less certain this would rather advocate for the minimax regret strategy. Also if there are indications that the two extremes donot have the same probabilities, the stochastic strategy would be more appropriate, because it takes into account the skewness of the distribution.

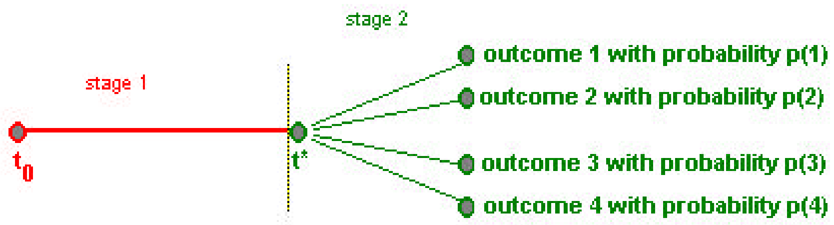
D. Uncertainty and Learning

When considering uncertainty it is important to take into account of the possibility of learning, because this can change the nature of the problem.

If there is uncertainty but no learning, the ideal policy can be set by minimising the expected costs for different possible states of nature, given the assumed probabilities for the given states of nature (or states of the world). The problem can be represented by a one-stage model.

If there is uncertainty and learning, learning will reduce or resolve the uncertainty. It is best to take account of this in the decision problem, because if more insight can be obtained after a number of periods, using this additional information in the model will improve the decision process.

Learning is a continuous process, but modelling it as such leads to complicated model. The easiest way of modelling learning is to consider two stages in the model. In the first stage there is uncertainty about the state of nature that will be realised in the second stage. At the start of the second stage (t*) the uncertainty is resolved and the true state of nature becomes known.



To better approximate the continuous process of learning, more stages can be introduced, however this complicates the model[5].

[5] If one thinks that the characteristic of learning being a continuous process is important for the problem, one can model this by introducing successive shorter periods in which uncertainty disappears gradually.

III. The Markal model

A. Introduction

Markal is a linear programming model for the representation of the energy system of a region. It considers the energy demand from the industry, the residential sector together with the commercial sector and the transport sector and the supply of the different energy vectors. It has 9 time periods of 5 years each. There are different versions of the Markal model, which characteristics are presented in annex. The version used for this project is "Stochastic Markal".

In "Stochastic Markal" the demand for energy services is exogenous[6]. The demand has to be satisfied by the supply, which is provided through an elaborated set of energy technologies. The objective of the model is to satisfy the demand at a minimal total cost, which includes investment -, operation - and maintenance costs. This is done by choosing the optimal mix of technologies, while satisfying constraints such as capacity limits, and peak-electricity constraints and eventual emissions constraints. The emissions of pollutants (CO2, NOx, SO2) by each technology are accounted for. Both annual and cumulative constraints can be placed on the emissions.

In the following section we give an algebraic presentation of the stochastic model. Thereafter we discuss some parameters and characteristics of the model, and their impact on the model.

B. Presentation of stochastic Markal

The **objective function** of stochastic Markal is based on the Mean-Variance model. It tries both to minimise the expected cost and the risk. The weight that is given to the risk is determined by the parameter λ. For a risk neutral decision maker λ equals 0, so that the only objective is to minimise the expected cost.

Stochastic Markal considers 2 stages:

1. in the first stage, t = 1$\rightarrow$ t2, there is uncertainty,
2. the second stage, t = t2 +1 $\rightarrow$ T, starts when this uncertainty is resolved and when the future becomes known; T is the last period in the model.

(i) Objective function: $\underset{X_t^i}{Min}$ (Z + λ* UPDEV);

where X_t^i are the decision variables, the investment in technologies

In stochastic MARKAL, the upside standard deviation (UPDEV) is introduced as the risk measure, there is no problem to consider the standard deviation, however both option leads to a non linear optimisation program, much more difficult to solve. At this stage, considering the size of the Belgian Markal model, λ has been set to 0 in the applications for this study, i.e risk neutrality is assumed.

[6] In Markal-micro and Markal-Ed the demand is determined inside the model, through the specification of demand functions which are depending on the price and thus on the marginal cost of the production.

(ii) with Z the expected total discounted system cost:

$$Z = \sum_i \left\{ \sum_{t=1}^{t2} C^i_t(x^i_t) * (1/1+r)^t + \sum_{s=1}^{S} prob._S * \left[\sum_{t=t2+1}^{T} * C^i_t(x^i_{t,s}) * (1/1+r)^t \right] \right\}$$

where $C^i_t(x^i_t)$ represents all the costs that can be attributed to the use of "technology i" at time period t. This cost includes fixed and variable costs.

$(1/1+r)^t$ is the discount factor that is used at period t.

$prob._S$ is the probability of state of nature s;

and Z_s the total discounted system cost for the stochastic strategy under outcome s can be computed as:

$$Z_S = \sum_i \{ \sum_{t=1\rightarrow T} C^i_t(x^i_{t,s}) * (1/1+r)^t \}.$$

(iii) the constraints, e.g.

- Useful energy demand constraint: a demand relation ensures that the end-use energy output is greater than or equal to the end-use demand that is specified by the user. And this for each demand sector (DM), time period (TP) and state of the world (SOW): $\sum_i x^i_{t,s}$ >= (exogenous) demand.
- Technology and capacity constraints.
- Periodical or cumulative constraints on CO_2 emissions may be imposed

C. Influence of parameter values and model characteristics on the solution generated by the model

Certain characteristics of the model and assumptions that are made in the model can influence the generated solution and it is important to have them in mind to get a better understanding of the model and also to get a better interpretation of the results.

There are points that are specific for *stochastic* Markal :

1. the year that uncertainty is resolved
2. the probabilities attached to the different possible outcomes and the degree of risk aversion
3. modelling of technologies in the model

and some characteristics relevant for all versions :

4. the discount rate
5. cumulative versus annual CO2-emission restrictions

1. The year that uncertainty is resolved

The date at which uncertainty is assumed to be resolved influences the solution. If uncertainty is resolved late, the stochastic will be closer to the deterministic strategy with the worst possible outcome. This is so because one of the assumptions of the model is that in the end it must be possible to satisfy the constraints for all possible outcomes. In the limiting case that the uncertainty is solved only at the end of the horizon, the stochastic path matches the deterministic path for the worst outcome.

2. The probabilities attached to the different possible outcomes and the degree of risk aversion

It is obvious that changing these parameters will result in changes of the solution. Therefore it might be important to make sensitivity studies around these parameters, as there is great uncertainty attached to them.

3. Modelling of technologies in the model

In the Belgian Markal model the fuel switch possibilities in the industry are mainly represented through technologies which can consume different types of fuels. Though the total cost is correct, the fuel switching possibilities and speed are overestimated. This can be important for the results, when using stochastic Markal model, because fuel switching technologies are very convenient in the uncertain stage. Ideally monofuel and bi- or trifuel technologies should be modelled explicitly and this will be taken into account in the future database for the Belgian Markal. This problem does not arise for the technologies for the residential and commercial, nor for the transport and electricity sector, where the different types of technologies are explicitly modelled.

4. The discount rate

The discount rate that is applied in the reference strategy of stochastic Markal equals 5% per year. The discount rate critically determines the comparison between the present and the future costs (and benefits). With a high discount rate, future expenditures (and benefits) have a smaller weight than current expenditures and therefore a large part of possible emission reduction will then be moved to the future.

5. Cumulative versus annual CO2-emission restrictions

Allowing for a cumulative CO2-emission limit instead of putting annual limits gives more leeway for solutions in the MARKAL model and will therefore result in a *less costly* solution. For the same reason the “banking” principle under the Kyoto-agreement is important. The Markal model itself chooses the most optimal path that corresponds with the requested cumulative emission constraint.

IV. Application of Stochastic Markal for Belgium : a comparison of stochastic strategy with deterministic strategies for a Kyoto scenario

A. Description of the Kyoto scenario

We consider two types of strategies for the Kyoto scenario:

1. deterministic strategies, one for each State of the World
2. a stochastic strategy combining the different States of the World

The model assumptions are:

1. In all the strategies, the CO2-emissions[7] for 1990 and 1995 are fixed to the observed levels[8].
2. In all strategies the CO2 emissions from 2008 to 2012 must be on average 7,5% below the CO2-emissions in 1990, this to satisfy the agreements under the Kyoto Protocol.
3. Four possible states of the nature are considered for the cumulative CO2 constraints to be imposed
4. Under the stochastic strategy, it is assumed that after 2012 it will become clear which one of 4 possible cumulative emission levels will have to be reached between 1990 and 2030. Therefore the first decision moment on which there is certainty concerning the state of nature is 2013. Thus, one path is followed until 2012 and starting from 2013, four different paths are possible, one for each alternative emission constraint.
5. Risk neutrality is assumed in the stochastic strategy.

The four possible states of nature, the attached cumulative emission levels (to be reached in 2030), and their attached probabilities are presented in the table below. The same cumulative emission constraints are used in the deterministic strategies, Det..., Det 0%, Det –8%, Det –25%.

State of nature cumulative emission level, to be reached in 2030		**Maximum on Cum. CO2-emissions between 1988 and 2032 (Million tons)**	**Probability State of Nature**
without cumulative constraint	Stoch. ...		0.25
stabilisation at 1990 level	Stoch. 0%	4541	0.25
-8% compared with stabilisation	Stoch. -8%	4178	0.25
-25%compared with stabilisation	Stoch. -25%	3406	0.25

For 1990, the Markal run gives a total of 100,912 Million tons CO2-emissions for Belgium. Therefore under the 0% cumulative reduction strategy, the average yearly emissions must be lower than or equal to 100,912 million tons. The total CO2 emissions between 1988[9] and 2032 (both years included) therefore equals 45 times 100,912 Million tons or 4.541 Million tons. Under the 8% cumulative reduction strategy, the average yearly CO2 emissions must be lower than or equal to 8% of the emissions in 1990, so they must be below 92,84 million tons. 45 times this amount equals 4.178. Under the -25% cumulative reduction strategy, the average yearly emissions are at 75,68

[7] In order to make the scenario congruent with reality we should not only put fixed bounds on CO2-emissions but also on the technologies that were used in the past period. This is not yet the case in this implementation.

[8] The figures for 1990 and 1995 are drawn from a Markal run under the scenario business as usual.

[9] Markal starts in 1988. The five year period indicated by its middle year 1990 stands for 1988, 1989, 2000, 2001 and 2002. “2030” stands for 2028, 2029, 2030, 2031, 2032.

million tons. The total cumulative emissions between 1988 and 2030 will therefore be lower than or equal to 3.406 million tons.)

B. CO_2 emissions and Costs

1. CO2 emission paths for the different scenarios

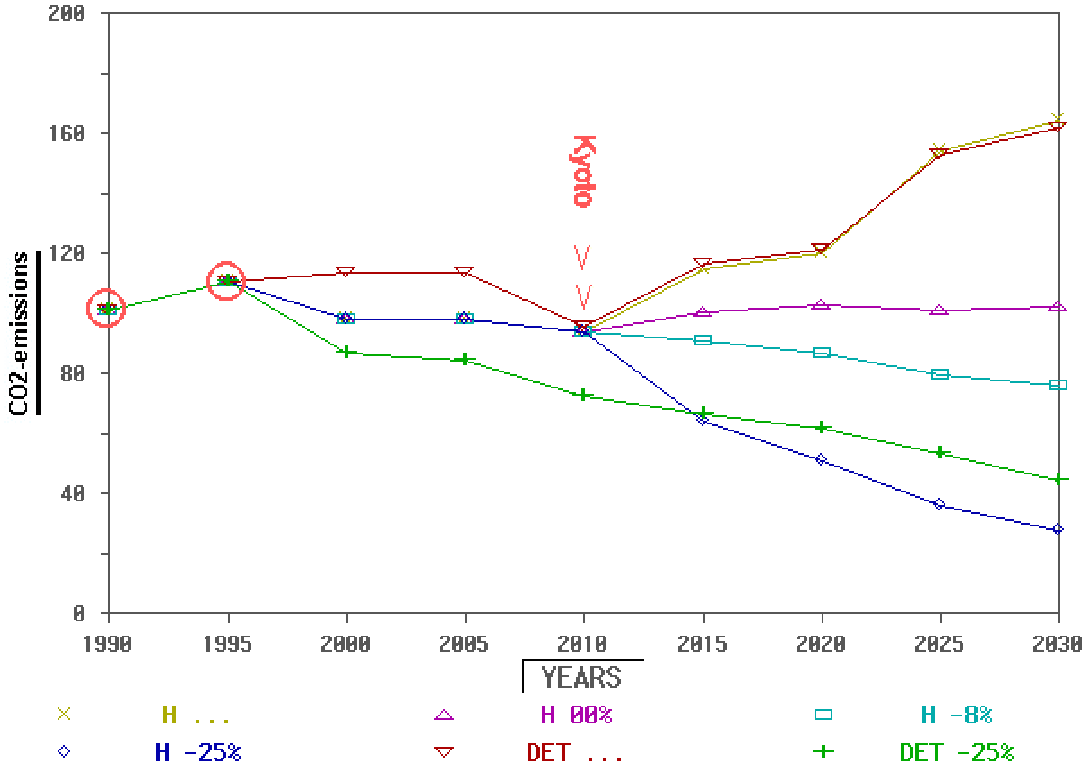
In the table and figure below, the CO2-emission paths of the stochastic strategy (Stoch..) and of two deterministic strategies (Det) are presented. As we mentioned already, the CO2-emissions for 1990 and 1995 were fixed. The stochastic strategy results in one path until the year 2012 (period 2010) and in four paths from the year 2013 (period 2015). The Kyoto constraints must be satisfied for all cases.

For strategy “Det -25%”, the CO2-emissions in 2010 are already lower than should according to the Kyoto-constraint. Therefore the Kyoto constraint is not binding under this strategy and does not result in extra costs.

Table 2: CO2 emission paths for Stochastic strategy, Det ... and Det -25% (M ton/year).

Strategy	1990	1995	2000	2005	2010	2015	2020	2025	2030
Stoch ...	100,91	110,79	98,41	97,84	93,34	114,35	120,18	153,78	163,48
DET ...	100,91	110,79	113,89	113,92	93,34	115,42	120,45	151,81	161,74
DET -25%	100,91	110,79	86,87	84,27	72,32	66,29	61,81	53,45	44,46

Figure 1: CO2 emission paths for different strategies.



2. Comparison of the costs for the different scenarios

Table 3 compares the total cost for the different scenarios, with the total discounted cost under "Det...", the "deterministic strategy without cumulative constraint", taken as a reference. *EV* stands for the Expected Value of the four stochastic cases.

Table 3: Total discounted costs and CO2 emissions under different strategies(/scenarios)

	Cost (MBF)	relativ cost 'Det ...' =100	Diff. in Cost with 'Det ...' (MBF)	CO2 emission (Mton)	relativ CO2 em. 'Det ...' =100
Stoch. ...	20745559	100,3	66362	5265	97,3
Stoch. -0%	20813974	100,7	134777	4541	83,9
Stoch. -8%	20985476	101,5	306279	4178	77,2
Stoch. -25%	22016738	106,5	1337541	3406	62,9
EV	*21140437*	*102*	*461240*	*4348*	80,3
Det ...	20679197	<u>100</u>	0	5411	<u>100</u>
Det -0%	20793734	100,6	114537	4541	83,9
Det -8%	20981696	101,5	302499	4178	77,2
Det -25%	21762620	105,2	1083423	3406	62,9

In order to shed more light on the advantages of the stochastic strategy, in Table 4 and Figure 2 below, we put together the costs that are incurred for the stochastic and deterministic strategy for the 4 possible outcomes after 2012. To explain Table 4 below, we remark that the first 4 cells of the header row give the 4 possible cumulative constraints, of which will be known from 2013 on which is the true one. So, for instance, for the row indicated by "Det..." the total discounted costs are represented for the deterministic case where one assumes until period 2010 that it is certain that there will be no limit on the cumulative CO2 emissions. If then indeed it will be revealed after period 2010 that there will be no limit on the emissions, the cost will be "100". However, if the limit on yearly CO2 emissions will be on average 0%, 8% or 25% lower than the 1990 level, the cost will be respectively 100.8, 102.0 and 108.7. In the last cell of this row the expected cost of the "Det..."- strategy is calculated as the sum of 0.25 times the cost under each of the four possible outcomes.

Table 4: Relative total discounted costs under different possible outcomes after 2012, for all strategies.

		POSSIBLE OUTCOMES				
		...	0	-8%	-25%	E.V.
St	Det...	**<u>100.0</u>**	100,8	102	108,7	102,88
ra	Det 0	100,2	**100,6**	101,6	107,3	102,42
te	Det -8%	100,3	100,7	**101,5**	106,8	102,34
gi	Det -25%	102,3	102,4	102,6	**105,3**	103,17
es	stochastic	100,4	100,7	101,6	106,6	102,33

For the stochastic scenario, the difference between the costs under the 'most extreme' outcomes remains limited. The cost of "Stoch. -25%" is 6% (or 1279 * 10^9 BF) higher than the cost of "Stoch...". In both cases we have the same (stochastic) strategy until 2012, but after 2012 in the first case a very strong cumulative constraint (-25%) has to be satisfied, whereas in the second case there is no cumulative constraint at all.

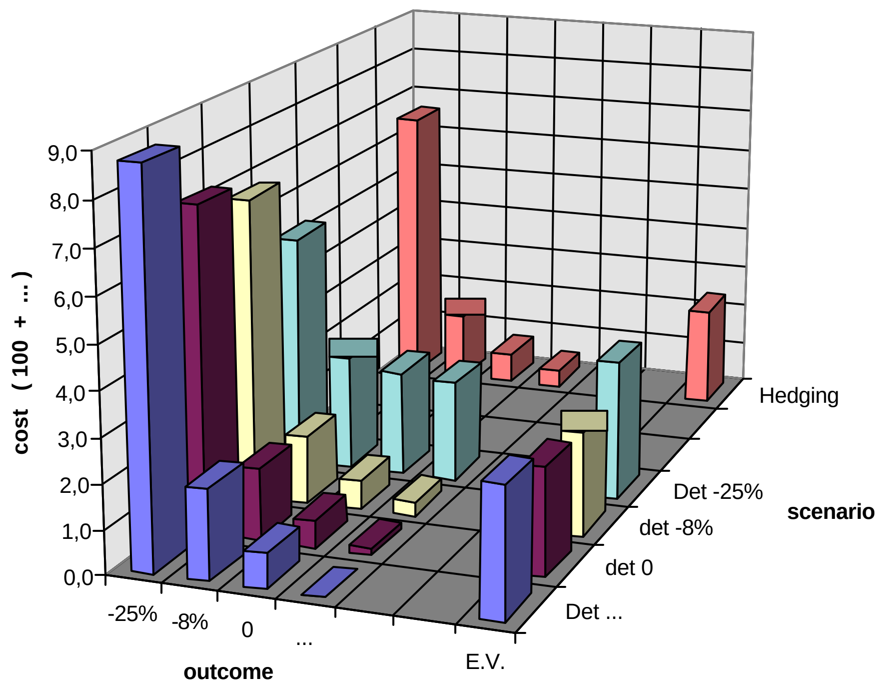
For each of the four possible cumulative CO2 restrictions that may be imposed after 2012, the total discounted system cost for the stochastic strategy never exceeds the cost of the "clairvoyant deterministic strategy with the correctly assumed outcome" with more than 1%.

If one looks at Table 4 one remarks that the total discounted system costs under the stochastic strategy are very close (+0.1%) to the best possible solutions for the "non- extreme" outcomes ("0%" and "-8%" as cumulative restriction). For the extreme outcomes they are further away from the best possible solution, 0.4% for outcome '...' and 1.3 % for outcome "-25%'.

If the solution of the stochastic strategy is compared with "Det 0%" and "Det -8%" for the four possible outcomes, the differences seem to be very small. Therefore it may seem that the stochastic strategy has little advantage over intermediate deterministic strategies and that it may not be worth the effort to use a stochastic strategy. This is not the case. First, since we deal with huge costs, a difference of 0.1% of the total discounted system cost remains a very large amount (i.e. $20.6 * 10^9$ BF). Secondly it is not always the case that the intermediate deterministic strategies are close to the stochastic strategy (cf. Richard Loulou and Amit Kanudia).

The graph below presents the same information as Table 4 above. Starting from each strategy (4 deterministic + 1 stochastic), for each of the 4 possible cumulative constraints, the part of the costs that exceed "100" are represented.

Figure 2: Total discounted costs under different possible outcomes after 2012, for all strategies.



In terms of the expected value, the stochastic strategy performs better than the deterministic strategies. This should be so, because the stochastic strategy is chosen in a way to generate the smallest possible expected total discounted cost.

For a certain outcome no solution can be less costly than the deterministic strategy which assumed from the start that this outcome was certain to take place. In Table 4 above these costs are presented in bold characters. In Table 5, we calculated the regret that can be incurred for each strategy and each outcome as a percentage of the total discounted system cost of "Det..." if the real outcome is "...". In Table 6, we calculated the same regret but now it is expressed in MBF.

Table 5: Regret for each strategy and each outcome as % of the total discounted system cost of "Det..." if the real outcome is "...".

		POSSIBLE OUTCOMES					
		...	0	-8%	-25%	E.V.	diff in E.V.
St	Det ...	**0**	0.2	0.5	3.4	102.88	0.55
ra	Det 0	0.2	**0**	0.1	2.0	102.42	0.09
te	Det -8%	0.3	0.1	**0**	1.5	102.34	0.01
gi	Det -25%	2.3	1.8	1.1	**0**	103.17	0.84
es	Stochastic	0.4	0.1	0.1	1.3	**102.33**	**0**

Table 6: Regret for each strategy and each outcome, in Million BF.

		POSSIBLE OUTCOMES					
		...	0	-8%	-25%	E.V.	diff in E.V.
St	Det ...	0	41324	103310	**702508**	21257054	113641
ra	Det 0	41324	0	20662	**413240**	21162009	18596
te	Det -8%	61986	20662	0	**309930**	21145480	2066
gi	Det -25%	**475226**	371916	227282	0	21316974	173561
es	Stochastic	82648	20662	20662	**268606**	21143413	0

From the 5 strategies presented in the table above, the stochastic strategy has the smallest maximum regret that can be incurred. The maximum regret that can be incurred under the stochastic strategy appears if outcome "-25%" is the true one. The regret is then 1,3% of the total discounted system cost of "Det..." if "..." is the true outcome or this is 269 * 10^9 BF. The minimax regret strategy itself was not calculated, but we can deduce that under the minimax regret strategy more effort would be taken to reduce CO2-emissions in the uncertain time-span than under the stochastic strategy. The maximum regret that can be incurred under "Det...", "Det 0" and "Det -8%" is respectively 3.4%, 2.0%, and 1.5%, and this always if outcome "-25%" appears to be the true one, so that it is a "future" regret. The maximum regret that can occur for strategy "Det -25%" is 2.3% and this if outcome "..." would be the true outcome, it is a "present" regret.

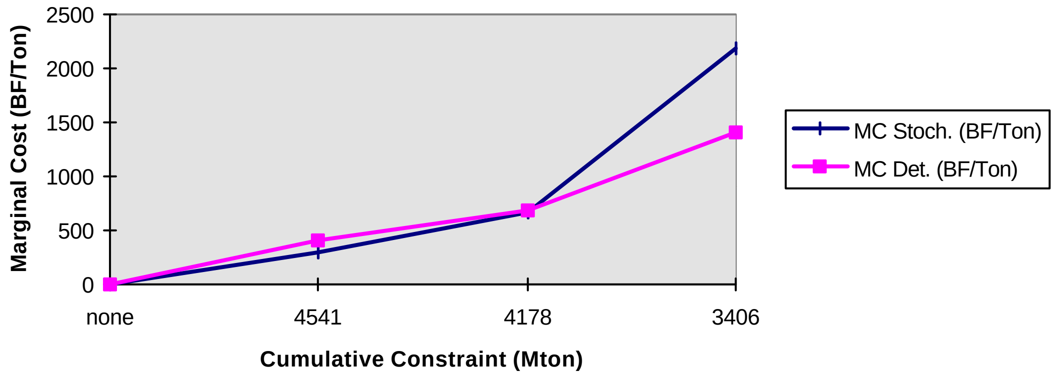
From the strategies above, it is strategy "Det -25%" that has the smallest maximum cost that can be incurred.

3. Trade-off between costs and CO2-emissions

In the table and graph below, the discounted marginal costs are presented for 4 possible cumulative constraints, both for the deterministic and the stochastic strategies.

Table 7 and Figure 3: Discounted marginal costs for stochastic and deterministic scenarios

constraint (relative to 1990)	constraint (Mton)	MC Stoch. (BF/Ton)	MC Det. (BF/Ton)
...	none	0	0
0%	4541	288	412
-8%	4178	668	677
-25%	3406	2192	1416



We see that the marginal cost for the cumulative constraint of 4541 Mton is smaller under "Stoch -0%" than under "Det -0%". This is normal, because in 2010 under "Stoch 0%" already more measures are taken to reduce the CO2-emissions than under "Det 0%", so that reducing the CO2-emissions with an extra Ton in comparison with the level of 4541 Mton is easier or less costly under "Stoch -0%" than under "Det -0%".

The marginal cost for the cumulative constraint of 3406 Mton is higher under "Stoch -25%" than under "Det -25%". This is so because in 2010 under "Stoch -25%" less measures were taken to reduce the CO2-emissions than under "Det -25%", so that limiting the CO2-emissions with an extra ton is more costly under "Stoch -25%" than under "Det -25%".

4. Influence of the Kyoto constraint on the generated solution

In order to look at the influence of the Kyoto constraint we performed runs in which the strategies are the same as before, with one exception: the Kyoto constraint[10] on the CO2-emission for the period 2010 was removed. These strategies will have a name similar to the strategy names we used before, but we add "NKC" in the strategy name, which stands for No Kyoto Constraint.

The CO2-emissions paths that are attached to the optimal solution for the different strategies without Kyoto constraint are presented in the first table below and the CO2-emission paths with the Kyoto constraint are presented in the second table below. We notice that the Kyoto constraint has no

[10] The Kyoto constraint in our model, limits the CO2-emissions for Belgium at a level of 95,5 Mton for the years 2008, 2009, 2010, 2011 and 2012.

effect on the strategy “Det -25%”. The Kyoto constraint is binding for the stochastic strategy, but the effect of this constraint is only very limited. The CO2-emission path for the stochastic strategy without the Kyoto constraint is only a very slightly higher than when this constraint is added. It has however a significant influence on the strategy “Det ...”.

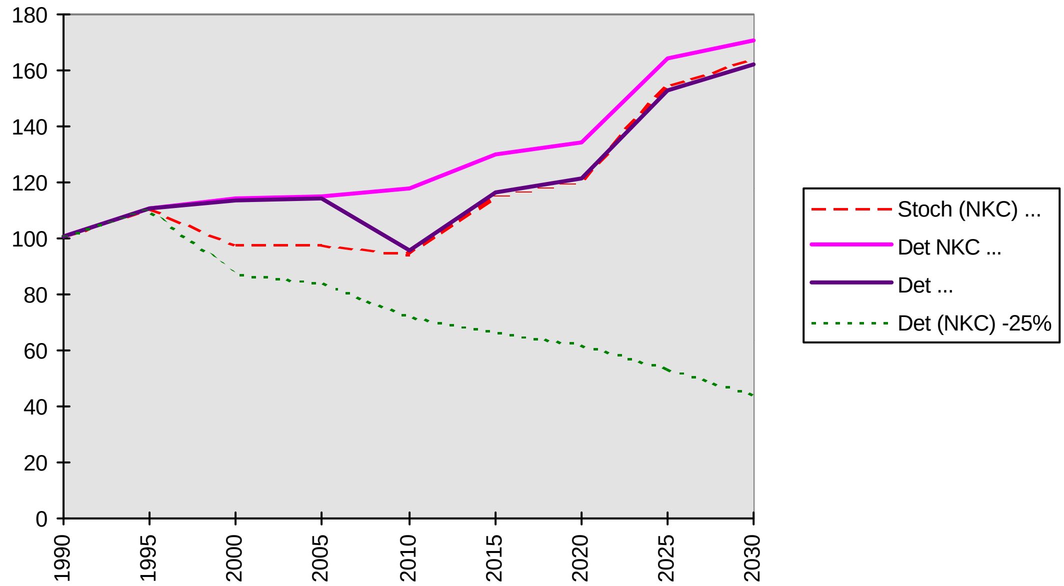
Table 8: CO2 emission paths for strategies WITHOUT the Kyoto Constraint (NKC)

Strategy	**1990**	**1995**	**2000**	**2005**	**2010**	**2015**	**2020**	**2025**	**2030**
Stoch NKC ...	100.9	110.8	98.1	97.9	94.3	114.7	120.5	154.1	164.1
DET NKC ...	100.9	110.8	114.5	114.7	117.5	129.9	134.0	164.4	171.0
DET NKC -25%	100.9	110.8	86.9	84.3	72.3	66.3	61.8	53.5	44.5

Table 9: CO2 emission paths for strategies WITH the Kyoto Constraint

Strategy	**1995**	**1990**	**2000**	**2005**	**2010**	**2015**	**2020**	**2025**	**2030**
Stoch ...	110.8	100.9	98.4	97.8	93.3	114.4	120.2	153.8	163.5
DET ...	110.8	100.9	*113.9*	*113.9*	*93.3*	*115.4*	*120.5*	*151.8*	*161.7*
DET -25%	110.8	100.9	86.9	84.3	72.3	66.3	61.8	53.5	44.5

Figure 4: CO2 emission paths, with and without Kyoto constraint.



This is also reflected in the total discounted cost, reproduced in *Table 10*.

Table 10: total discounted system cost for strategies with and without Kyoto constraint

strategy	**Stoch ...**	**Det ...**	**Det -25%**	**Stoch NKC ...**	**Det NKC ...**	**Det NKC -25%**
Cost (MBF)	20745559	2.1E+07	21762620	20742618	20576834	21762620
Cost (relativ)	100.32	100.00	105.24	100.31	99.50	105.24

C. The primary energy demand

The primary energy demand under the different strategies are reproduced in Table 11 in PJ and in Table 12 the primary energy demand is expressed relative to strategy “Det ...”.

The main points are:

- under “Det...” and “Det -25%”, there is a shift away from solids and liquids towards gas and nuclear energy, when the CO2-constraint becomes more stringent. In the det… scenario this shift is observed only in 2010 when the Kyoto-constraint has to be satisfied[11].
- under the stochastic strategy the same shifts occur from 2000 onwards, however with a smaller increase in nuclear until 2010. The full potential of nuclear is only used after 2010 when a CO2 constraint appears to be necessary. A balance between gas and nuclear energy seems a hedging strategy to face eventual CO2 constraints after 2010.

On total the primary energy demand in the stochastic scenario decreases slightly (- 2% in 2010) compared to “Det...”, whereas the primary energy demand under “Det -25%” decreases by 7% and 13% respectively in 2010 and 2030 in comparison to “Det...”. This is mainly due to conservation and to the greater efficiency in the use of gas compared to solids and liquid fuels. The stochastic strategy lies rather in between the 2 deterministic scenarios. This can be explained by the probability chosen (0.25 for each scenario) and by the relative easiness to switch between fuels in the Belgian Markal model. In the year 2010, strategy “Det ...” gets closer to the stochastic strategy compared to the previous periods, but this can entirely be explained by the Kyoto constraint.

On Table 11 and Table 12 the paths of the use of the primary energy sources for the 2 deterministic strategies are presented. For each of the 6 energy sources there is also a graph that gives the consumption path for the different strategies between 1990 and 2030, as well as a table.

Table 11: Primary energy use per strategy in PJ

strategy /CASE	E-source	1990	1995	2000	2005	2010	2015	2020	2025	2030		% of total 1990	% of total 2010	% of total 2030
stoch.	RENEW	8	8	22	24	27						0%	1%	
stoch.	NUCLEAR	382	396	446	446	499						20%	23%	
stoch.	FOS SOLID	388	438	226	218	160						20%	8%	
stoch.	FOS GAZ	301	284	439	501	641						16%	30%	
stoch.	FOS LIQ	850	932	918	884	808						44%	38%	
stoch.	CONSERV	182	176	224	262	275						9%	13%	
stoch.	Total (*)	1929	2058	2051	2074	2135						100%	100%	
Det ...	RENEW	8	8	8	8	27	23	23	23	23		0%	1%	1%
Det ...	NUCLEAR	382	396	446	446	521	416	420	140	140		20%	24%	6%
Det ...	FOS SOLID	397	439	392	312	152	304	320	652	710		21%	7%	29%
Det ...	FOS GAZ	282	257	301	386	592	454	473	427	398		15%	27%	16%
Det ...	FOS LIQ	853	950	1021	1063	889	1060	1095	1130	1206		44%	41%	49%
Det ...	CONSERV	174	176	183	191	268	241	232	235	232		9%	12%	9%
Det ...	Total (*)	1922	2051	2168	2216	2181	2257	2330	2371	2476		100%	100%	100%
Det -25%	RENEW	8	8	22	25	28	35	39	45	48		0%	1%	2%
Det –25%	NUCLEAR	382	396	446	446	557	557	557	557	557		20%	27%	26%
Det –25%	FOS SOLID	398	395	131	74	20	6	4	4	4		21%	1%	0%
Det –25%	FOS GAZ	334	454	607	707	768	829	863	905	935		17%	38%	43%

[11] Without Kyoto constraint there is no increase in the nuclear capacity.

Det –25%	FOS LIQ	813	860	763	730	658	616	618	618	608		42%	32%	28%
Det –25%	CONSERV	174	189	262	299	318	336	346	358	368		9%	16%	17%
Det –25%	Total (*)	1934	2112	1970	1982	2030	2043	2081	2129	2152		100%	100%	100%

(*) The sum here does not include the conservation of energy; source: table Primary in MUSS.

The difference in primary energy use of the strategies "stoch." and "Det -25%" with strategy "Det ..." are represented below, as a percentage of the use under strategy "Det ...". e.g. under strategy "stoch.", in the year 2000, 170% more renewables is used than under "Det ...".

Table 12: Primary energy use: relative.

strategy /CASE	E-source	1990	1995	2000	2005	2010	2015	2020	2025	2030		% of total 1990	% of total 2010	% of total 2030
stoch.	RENEW	0%	0%	170%	201%	0%						0%	2%	
stoch.	NUCLEAR	0%	0%	0%	0%	-4%						0%	-2%	
stoch.	FOS SOLID	-2%	0%	-42%	-30%	6%						-3%	8%	
stoch.	FOS GAZ	7%	10%	46%	30%	8%						6%	11%	
stoch.	FOS LIQ	0%	-2%	-10%	-17%	-9%						-1%	-7%	
stoch.	CONSERV	4%	0%	22%	37%	3%						4%	5%	
stoch.	Total (*)	0%	0%	-5%	-6%	-2%								
Det -25%	RENEW	0%	0%	170%	205%	3%	56%	70%	97%	111%		-1%	10%	143%
Det -25%	NUCLEAR	0%	0%	0%	0%	7%	34%	33%	299%	299%		-1%	15%	359%
Det -25%	FOS SOLID	0%	-10%	-67%	-76%	-87%	-98%	-99%	-99%	-99%		-1%	-86%	-99%
Det -25%	FOS GAZ	19%	76%	102%	83%	30%	83%	82%	112%	135%		18%	39%	170%
Det -25%	FOS LIQ	-5%	-9%	-25%	-31%	-26%	-42%	-44%	-45%	-50%		-5%	-20%	-42%
Det -25%	CONSERV	0%	7%	43%	57%	19%	39%	49%	52%	59%		-1%	28%	83%

(*) The sum here does not include the conservation of energy

It can be observed that for the same energy sources, there are some differences in the demand for energy in the year 1990 (and 1995) for different strategies. This is due to the fact that for these years, though the CO2-emissions where fixed at the BUS levels, the technologies were not. In order to get a more realistic prediction, it would be better to fix the technologies as well. This was not done because on this moment it is difficult to fix them in Markal and because the differences between the scenarios remain marginal this is not really a problem.

Figure 5: Source of "Primary energy input" for strategy det ...

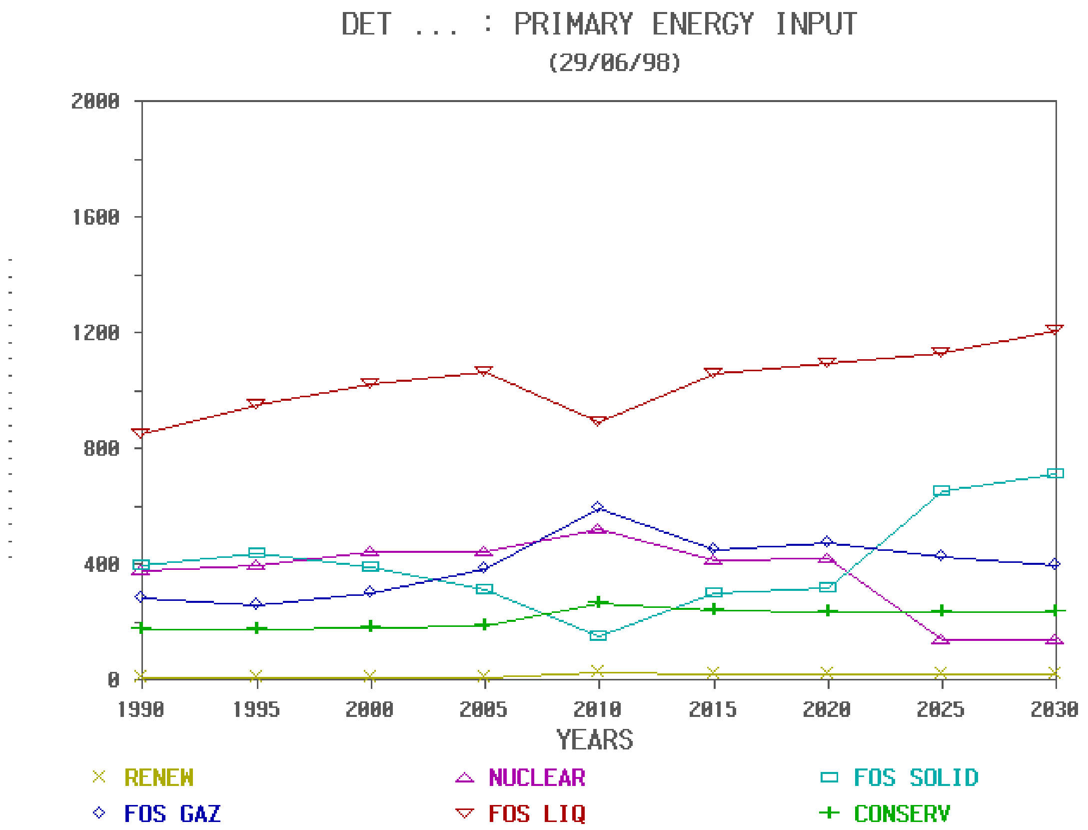


Figure 6: Source of "Primary energy input" for strategy det -25%

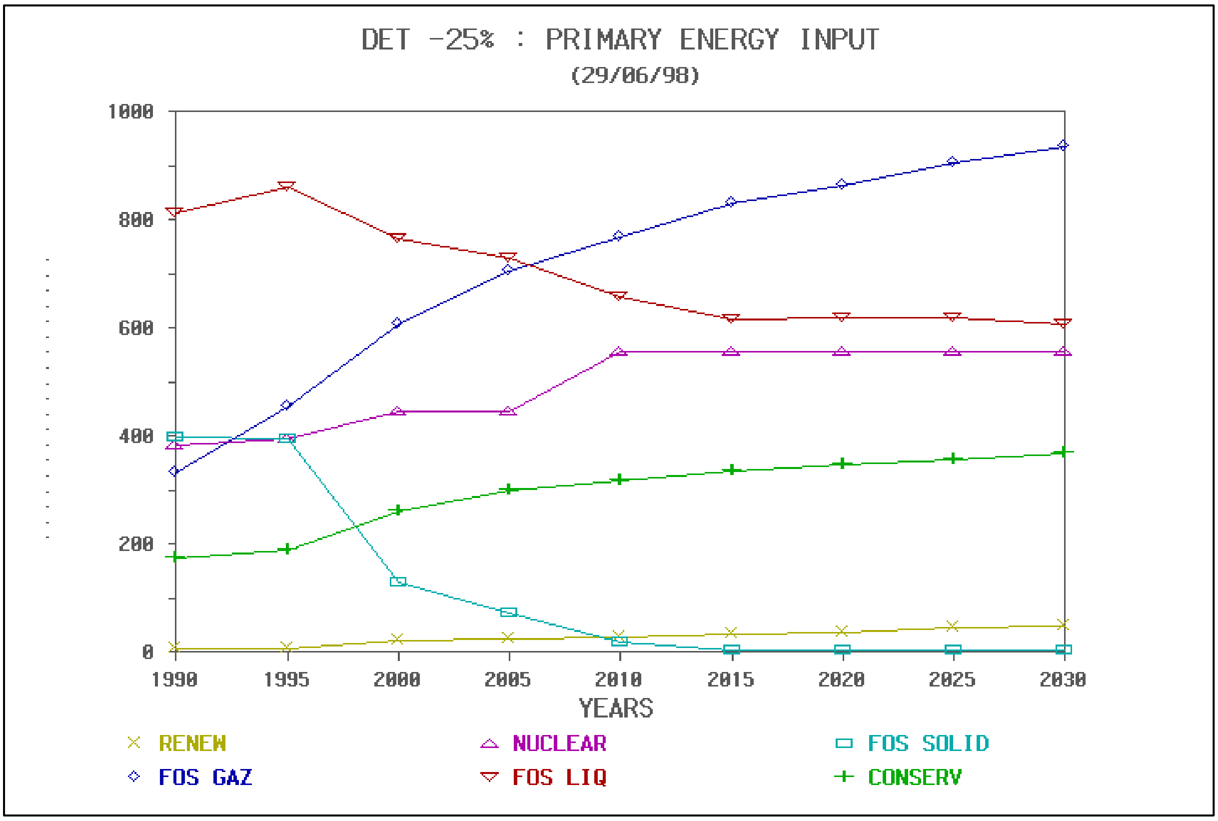
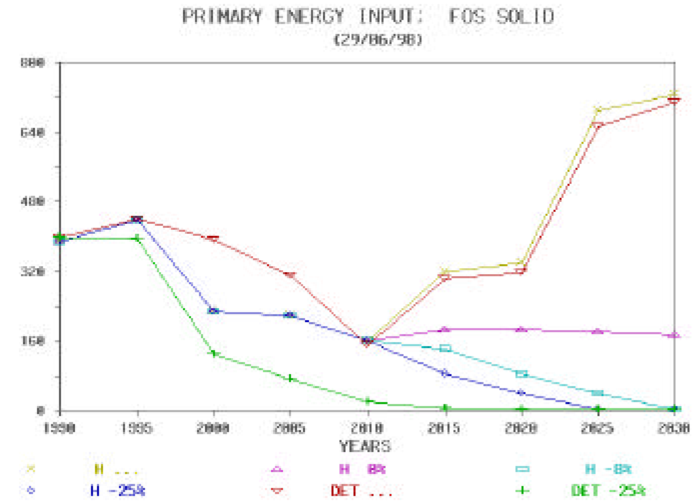


Figure 7, figure 8 and figure 9:



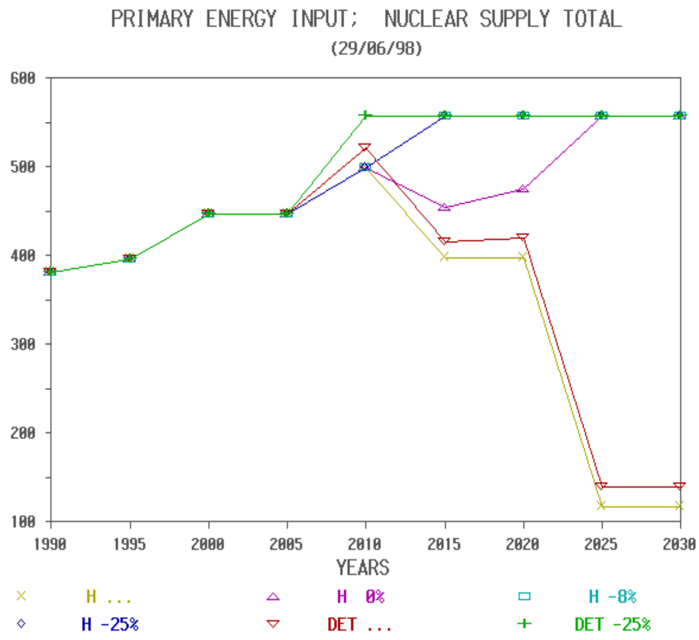
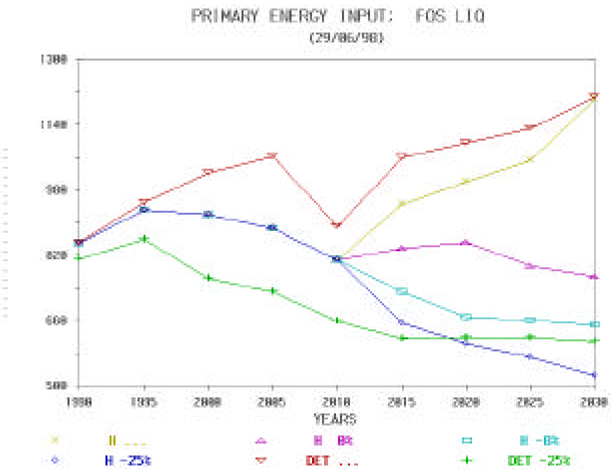
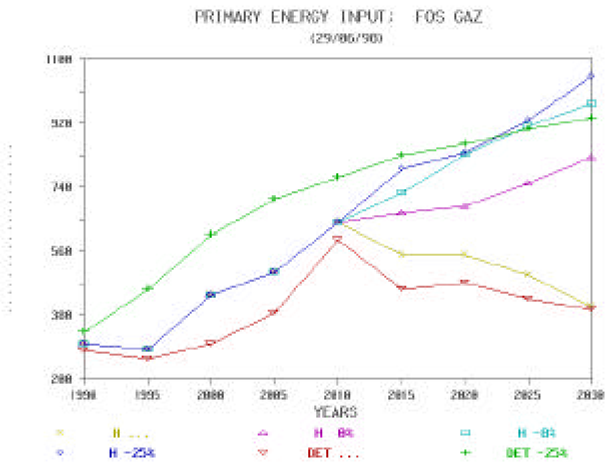
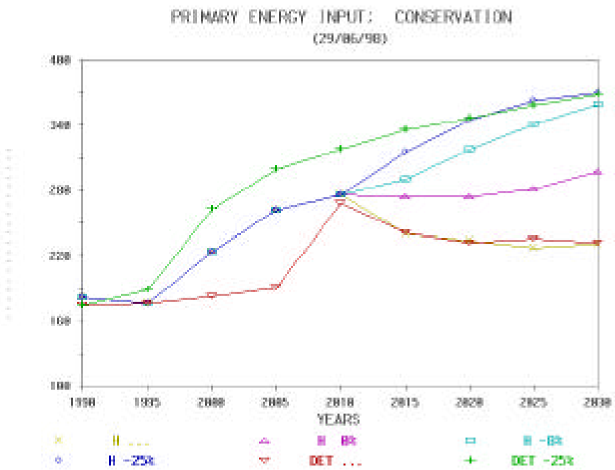


Figure 10, figure 11 and figure 12:







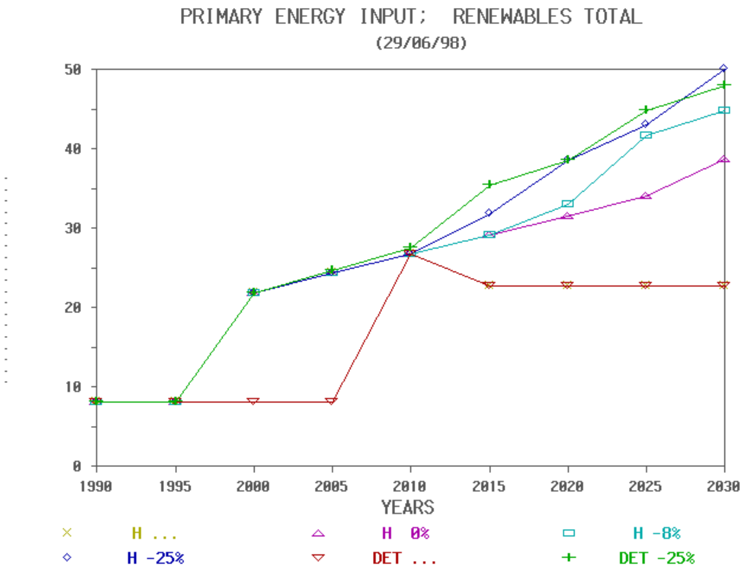

PRIMARY ENERGY INPUT; RENEWABLES TOTAL
(29/06/98)
50
40
30
20
10
0
1990
1995
2000
2005
2010
2015
2020
2025
2030
YEARS
H ...
H 0%
H -8%
H -25%
DET ...
DET -25%

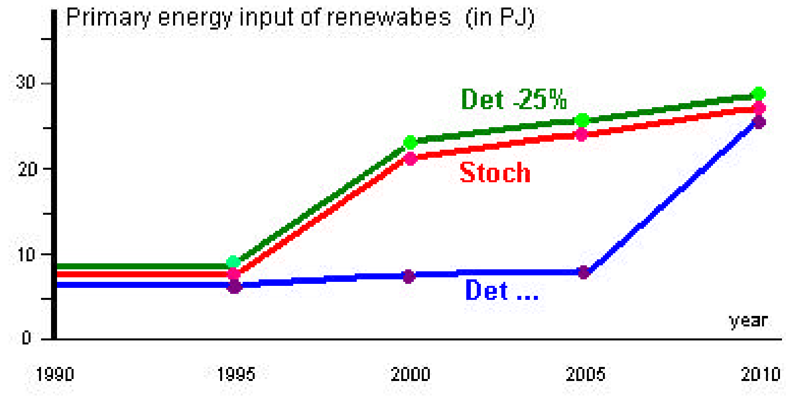
Table 13: Use of energy sources under the different strategies.

Case	Energy source	1990	1990(*)	2010	2010(*)	2030	2030(*)
STOCH. ...	RENEW	8.1	1.00	26.8	1.00	22.7	1
STOCH. 0%	RENEW	8.1	1.00	26.8	1.00	38.7	1.7
STOCH. -8%	RENEW	8.1	1.00	26.8	1.00	44.8	1.97
STOCH. -25%	RENEW	8.1	1.00	26.8	1.00	50.0	2.2
DET ...	RENEW	8.1	1.00	26.8	1.00	22.7	1
DET -25%	RENEW	8.1	1.00	27.5	1.03	47.9	2.11
STOCH. ...	NUCLEAR	381.9	1.00	498.8	0.96	116.9	0.84
STOCH. 0%	NUCLEAR	381.9	1.00	498.8	0.96	557.0	3.99
STOCH. -8%	NUCLEAR	381.9	1.00	498.8	0.96	557.0	3.99
STOCH. -25%	NUCLEAR	381.9	1.00	498.8	0.96	557.0	3.99
DET ...	NUCLEAR	381.9	1.00	521.4	1.00	139.5	1
DET -25%	NUCLEAR	381.9	1.00	557.0	1.07	557.0	3.99
STOCH. ...	FOS SOLID	388.3	0.98	160.3	1.06	725.8	1.02
STOCH. 0%	FOS SOLID	388.3	0.98	160.3	1.06	174.2	0.25
STOCH. -8%	FOS SOLID	388.3	0.98	160.3	1.06	4.4	0.01
STOCH. -25%	FOS SOLID	388.3	0.98	160.3	1.06	4.4	0.01
DET ...	FOS SOLID	397.1	1.00	151.8	1.00	710.0	1
DET -25%	FOS SOLID	397.7	1.00	19.6	0.13	4.4	0.01
STOCH. ...	FOS GAZ	300.6	1.07	641.2	1.08	405.2	1.02
STOCH. 0%	FOS GAZ	300.6	1.07	641.2	1.08	823.6	2.07
STOCH. -8%	FOS GAZ	300.6	1.07	641.2	1.08	973.7	2.45
STOCH. -25%	FOS GAZ	300.6	1.07	641.2	1.08	1052.4	2.65
DET ...	FOS GAZ	281.6	1.00	592.3	1.00	397.7	1
DET -25%	FOS GAZ	333.7	1.19	768.2	1.30	935.0	2.35
STOCH. ...	FOS LIQ	850.1	1.00	808.1	0.91	1202.6	1
STOCH. 0%	FOS LIQ	850.1	1.00	808.1	0.91	768.7	0.64
STOCH. -8%	FOS LIQ	850.1	1.00	808.1	0.91	647.5	0.54
STOCH. -25%	FOS LIQ	850.1	1.00	808.1	0.91	526.2	0.44
DET ...	FOS LIQ	853.1	1.00	889.0	1.00	1206.3	1
DET -25%	FOS LIQ	813.0	0.95	658.0	0.74	608.0	0.5
STOCH. ...	CONSERV	182.2	1.04	275.3	1.03	231.5	1
STOCH. 0%	CONSERV	182.2	1.04	275.3	1.03	297.2	1.28
STOCH. -8%	CONSERV	182.2	1.04	275.3	1.03	369.1	1.55
STOCH. -25%	CONSERV	182.2	1.04	275.3	1.03	369.9	1.6
DET ...	CONSERV	174.4	1.00	267.7	1.00	231.9	1
DET -25%	CONSERV	174.4	1.00	318	1.19	368	1.59
STOCH. ...	TOTAL	2310.9	1.00	2634	0.97	2590.1	0.99
STOCH. 0%	TOTAL	2310.9	1.00	2634	0.97	2919.2	1.12
STOCH. -8%	TOTAL	2310.9	1.00	2634	0.97	2784.4	1.06
STOCH. -25%	TOTAL	2310.9	1.00	2634	0.97	2747	1.05
DET ...	TOTAL	2303.7	1.00	2702.7	1.00	2615.7	1.00
DET -25%	TOTAL	2316.3	1.01	2587.3	0.96	2709.3	1.04

(*) The figures in this column are relative to the demand for energy for the case DET ...

It is interesting to note that the stochastic path does not lie necessarily between the deterministic strategies: for some characteristics it may lie closer to one extreme deterministic path and for others it may lie closer to the other or even for the same characteristic for some periods it may lie closer to the one and for other time periods it may lie closer to the other. This can be illustrated by looking at the primary energy use of renewables, gas and solids in the previous tables. On Figure 13 we see that the stochastic path for the use of renewables lies very close to the extreme deterministic path "Det -25%", while the stochastic path for the use of gaseous fossil fuels lies more closely to the other extreme deterministic path, "Det...". On Figure 14, we represented the stochastic path for the use of solid fossil fuels. We see that the stochastic path lies very close to "Det ..." in 1995, while in 2000 it lies more closely to "Det -25%" and in 2010 it lies again more closely to "Det ...".

Figure 13: Primary energy input of renewables and gaseous fossil fuels (in PJ)



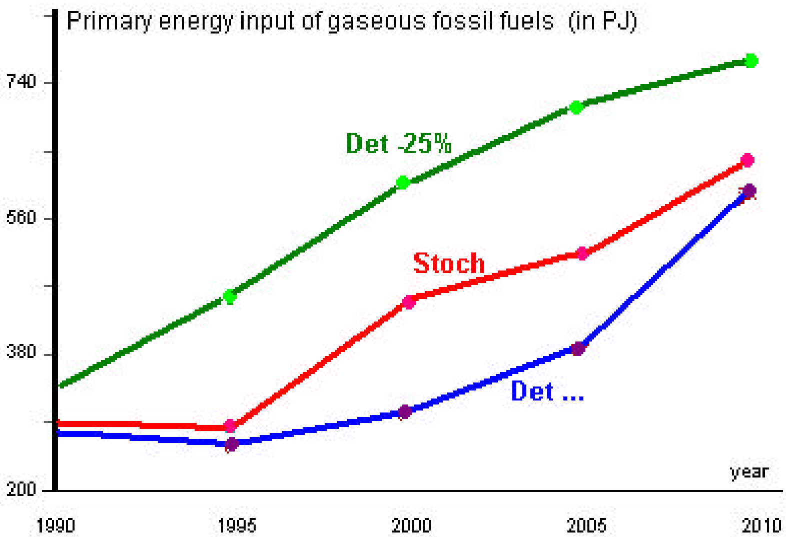
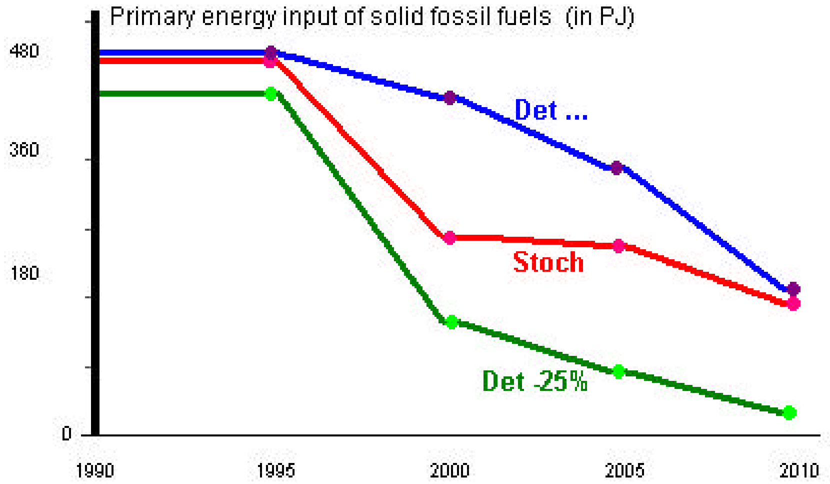


Figure 14: Primary energy use of solid fossil fuels (in PJ)



D. *Final Energy Demand*

The graph and tables below show that the final energy demand is decreasing in the industry and in the residential and commercial sectors when CO2 constraints are imposed, both in the stochastic and the deterministic scenarios. No decrease is observed in the transport sector, with the exception of the end of the horizon under the most stringent CO2 scenario.

Table 14: Energy use for the different strategies in industrial, residential and transport sector. (For "Det ..." in PJ; for the other strategies relative to "Det ...")

scenario	sector	1990	1995	2000	2005	2010	2030
Stoch.	INDUSTRY	-2,4%	-0,2%	-8,8%	-10,7%	-1,7%	
Stoch.	RESIDENTIAL	-0,2%	0,0%	-2,6%	-6,5%	-3,7%	
Stoch.	TRANSPORT	0,0%	0,0%	0,0%	0,0%	0,0%	
Stoch.	TOTAL	-0,8%	-0,1%	-4,0%	-6,0%	-1,8%	
Det ...	INDUSTRY	493	569	613	629	580	692
Det ...	RESIDENTIAL	487	513	547	573	564	703
Det ...	TRANSPORT	484	502	523	542	562	582
Det ...	TOTAL	1464	1586	1681	1744	1705	1977
Det -25%	INDUSTRY	0,0%	-3,9%	-15,0%	-20,0%	-13,3%	-24,0%
Det -25%	RESIDENTIAL	0,0%	0,0%	-7,1%	-9,6%	-15,6%	-44,8%
Det -25%	TRANSPORT	0,0%	0,0%	0,0%	0,0%	0,0%	0,2%
Det -25%	TOTAL	0,0%	-1,5%	-7,7%	-10,4%	-9,7%	-24,3%

(the energy use for strategy "Det ..." is expressed in PJ ; the difference in energy use under the other strategies is presented relative to the use under "Det ...")

As the demand for energy services is fixed in this version of Markal, the decrease is due to a more efficient use of energy (through shifting towards gas) and through conservation. In the stochastic Markal scenario, these shifts are already starting in 2000.

In the industrial sector, gas is clearly a hedging strategy until 2010, substituting oil and coal. There is an increase in conservation in the stochastic scenario compared to “Det ...”, but it remains far below the conservation under “Det -25%”.

In the residential and commercial sector, the same trend is observed for the substitution of oil with gas.

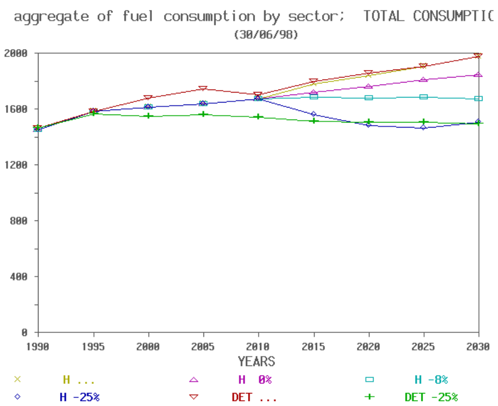


Figure 15: Total energy consumption

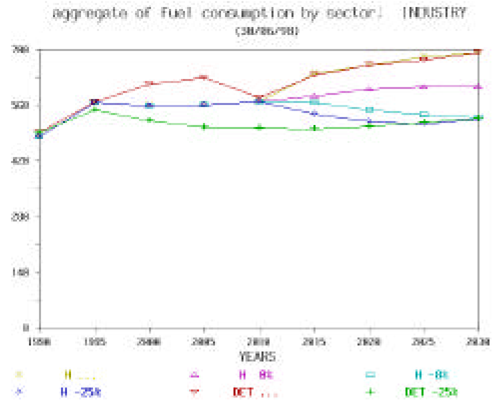


Figure 17: Energy consumption in industry

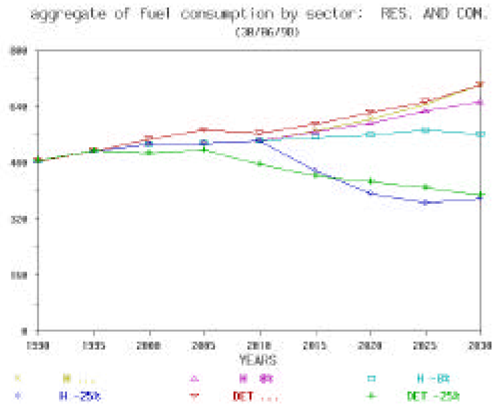


Figure 16: Energy consumption in residential and commercial sector

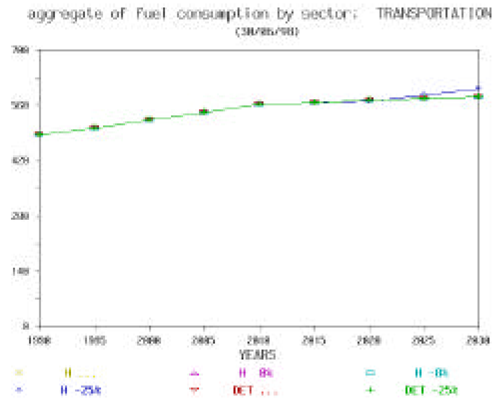


Figure 18: Energy consumption for transport

Table 15: Final energy demand in the industry (PJ)

Strategy		1990	1995	2000	2005	2010	2015	2020	2025	2030
Stoch ...	TOTAL	487	579	563	567	**576**	649	670	691	701
Det ...		500	580	623	639	**587**	647	670	683	701
Det -25%		501	555	498	480	**477**	473	479	519	528
Stoch ...	Energy conservation	11	18	22	32	**41**	41	33	31	31
Det ...		12	18	22	26	**39**	43	33	34	31
Det -25%		12	15	39	67	**77**	88	95	102	108
Stoch ...	I:Coal + coke	170	209	171	170	**129**	214	214	214	214
Det ...		182	209	219	218	**119**	214	214	214	214
Det -25%		183	189	89	54	**18**	5	4	4	4
Stoch ...	I:OIL	99	120	63	20	**11**	118	125	126	128
Det ...		101	134	137	126	**11**	117	125	126	128
Det -25%		101	106	16	10	**11**	11	11	11	11
Stoch ...	I:NATURAL GAS	99	113	140	179	**220**	96	99	101	101
Det ...		99	104	112	98	**239**	94	99	100	101
Det -25%		99	107	210	210	**226**	227	228	260	261
Stoch ...	I:HYDROGEN	0	0	0	0	**0**	0	0	0	0
Det ...		0	0	0	0	**0**	0	0	0	0
Det -25%		0	0	0	0	**0**	12	18	13	0

Stoch ...	Electricity	116	130	150	154	**171**	176	186	203	212
Det ...		115	130	144	157	**173**	176	186	197	212
Det -25%		115	133	146	162	**181**	186	192	199	207
Stoch ...	I:HEAT	0	3	36	41	**42**	42	43	43	44
Det ...		0	0	8	37	**42**	42	43	43	44
Det -25%		0	17	62	70	**69**	61	56	31	44

Table 16: Final energy demand in the residential and tertiary sector (PJ)

Strategy		**1990**	**1995**	**2000**	**2005**	**2010**	**2015**	**2020**	**2025**	**2030**
Stoch ...	TOTAL	486	514	533	535	**542**	571	606	648	704
Det ...		487	514	547	573	**563**	591	624	654	703
Det -25%		487	513	508	518	**476**	443	426	405	385
Stoch ...	Energy saving by insulation	66	76	86	111	**121**	128	135	141	146
Det ...		66	76	84	94	**120**	127	134	141	146
Det -25%		66	76	101	114	**128**	141	150	161	170
Stoch ...	RT:COAL	22	18	13	10	**6**	9	14	12	4
Det ...		22	21	16	14	**6**	9	14	12	4
Det -25%		22	18	13	9	**2**	0	0	0	0
Stoch ...	RT:OIL	216	256	275	258	**150**	164	204	248	383
Det ...		217	260	298	316	**266**	274	295	321	386
Det -25%		179	202	181	137	**48**	3	0	0	0
Stoch ...	RT:NATURAL GAS	162	155	161	161	**242**	244	225	209	127
Det ...		161	149	143	136	**160**	160	158	144	124
Det -25%		200	207	221	256	**259**	242	196	137	77
Stoch ...	RT:HEAT	0	2	8	20	**41**	44	47	49	49
Det ...		0	1	6	17	**37**	43	47	49	49
Det -25%		0	5	11	24	**44**	48	51	53	54
Stoch ...	RT:ELECTRICITY	80	81	76	85	**103**	109	115	129	139
Det ...		80	81	82	90	**94**	104	111	128	140
Det -25%		80	81	82	91	**121**	149	178	215	253

If one looks at the technology level, for most technologies the use under the stochastic strategy lies in between the use of the extreme deterministic solutions. But there are examples where this is not so. The optimal use of a technology under the stochastic strategy may exceed the use of both deterministic strategies, when this technology allows for an easy adaptation for the different possible outcomes after 2012. There are two such examples in our scenarios: "the use of natural gas in gas boilers in the industry" and "the use of natural gas for boilers in the residential sector", as can be seen in the tables below.

Table 17: The use of natural gas boilers in the industry in PJ (GI1.IPA + GI1.IYA)

Scenario	1990	1995	2000	2005	2010	sum(*)	2015(°)	2020(°)
STOCH.	22,8	24,2	45,5	44,4	41,8	179	0,1	0,1
DET ...	22,8	15,2	16,7	0,1	45,6	100	0,1	0,1
Det NKC ...	22,8	15,2	7,7	0,1	0,1	46	0,1	0,1
DET -25%	22,8	18,2	8,9	1,4	4,6	56	0,1	0,3

(*) sum from 1990 to 2010.

Table 18: The use of natural gas boilers in the residential sector in PJ (GR1.R1E)

Scenario	1990	1995	2000	2005	2010	sum(*)	2015(°)	2020(°)
STOCH.	1,5	1,1	5,8	4,9	49,9	63	49,9	47,6
DET ...	1,5	1,2	1	0,1	0,1	4	0,1	0,1
Det NKC ...	1,5	1,2	1	0,1	0,1	4	0,1	0,1
DET -25%	1,5	1,2	8,1	21	21	53	21	13,9

(*) sum from 1990 to 2010.

E. The electricity sector

Under the stochastic strategy both the nuclear and gas options are present, as shown in the tables below. The nuclear potential is not fully used, contrary to "Det -25%". The increase in the use of nuclear energy under "Det..." is entirely due to the Kyoto constraint in 2010. Because under "Det..." in 2010 more energy is produced by nuclear energy, less energy has to be produced by the gas-fired STAG power plants.

Table 19: Production of electricity in nuclear power plants (LWR.E21) under the different strategies (PJ)

SCEN.	1990	1995	2000	2005	2010	sum(*)	2015(°)	2020(°)
STOCH.	147	152	172	172	192	835	153	153
DET ...	147	152	172	172	201	844	160	162
Det NKC ...	147	152	172	172	172	815	133	133
DET -25%	147	152	172	172	214	858	214	214

(*) sum of energy 1990,1995,2000,2005 and 2010.
(°) After 2012 there is no more stochastic path, the numbers are those for "Stoch. ..."

Table 20: Production of electricity with gas in Stag power plants and in decentralised Stags for combined heat and power under the different strategies (in PJ)

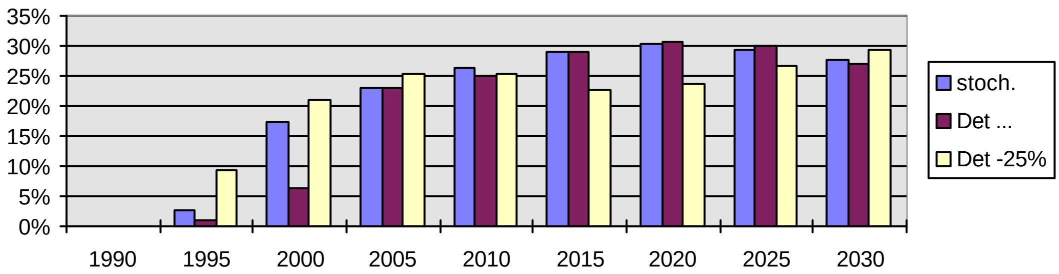
Scenario	1990	1995	2000	2005	2010	sum(*)	2015(°)	2020(°)
STOCH.	0	18,07	45,46	34,07	28,95	127	54,7	52,2
DET ...	0	27,84	32,73	37,66	13,76	112	48,4	46,2
Det NKC ...	0	27,76	32,89	37,2	60,76	159	46,13	45,69
DET -25%	0	5,4	25,33	40,63	36,53	108	98,7	142,4

The shift from centralised to decentralised electricity production starts a bit earlier under the stochastic strategy and under "Det -25%", but by 2010 the shares under the different strategies are relatively close to each other.

Table 21 and Figure 19: Decentralised production as a percentage of the total electricity production

	1990	1995	2000	2005	2010	2015	2020	2025	2030
stoch.	0%	3%	17%	23%	26%	29%	30%	29%	28%

Det ...	0%	1%	6%	23%	25%	29%	31%	30%	27%
Det -25%	0%	9%	21%	25%	25%	23%	24%	27%	29%



The disposal of CO2 in aquifers is only used when high CO2-emissions are imposed. It is also used under “Det...” in 2010 to satisfy the Kyoto constraint.

Table 22: Disposal of CO2 in aquifers.

Description of technology	SCEN.	1990	1995	2000	2005	2010	2015	2020	2025	2030	sur
Carbon dioxide disposal in aquifers	STOCH.	0	0	0	0	0	0	0	0	0	
(CDR.SCZ)	**DET ...**	0	0	0	0	**1,21**	0	0	0	0	
	Det NKC ...	0	0	0	0	0	0	0	0	0	
	DET -25%	0	0	0,01	0,01	**4,58**	**9,45**	**15,68**	**24,35**	**28,89**	8

(*) sum of energy 1990, ...2030.

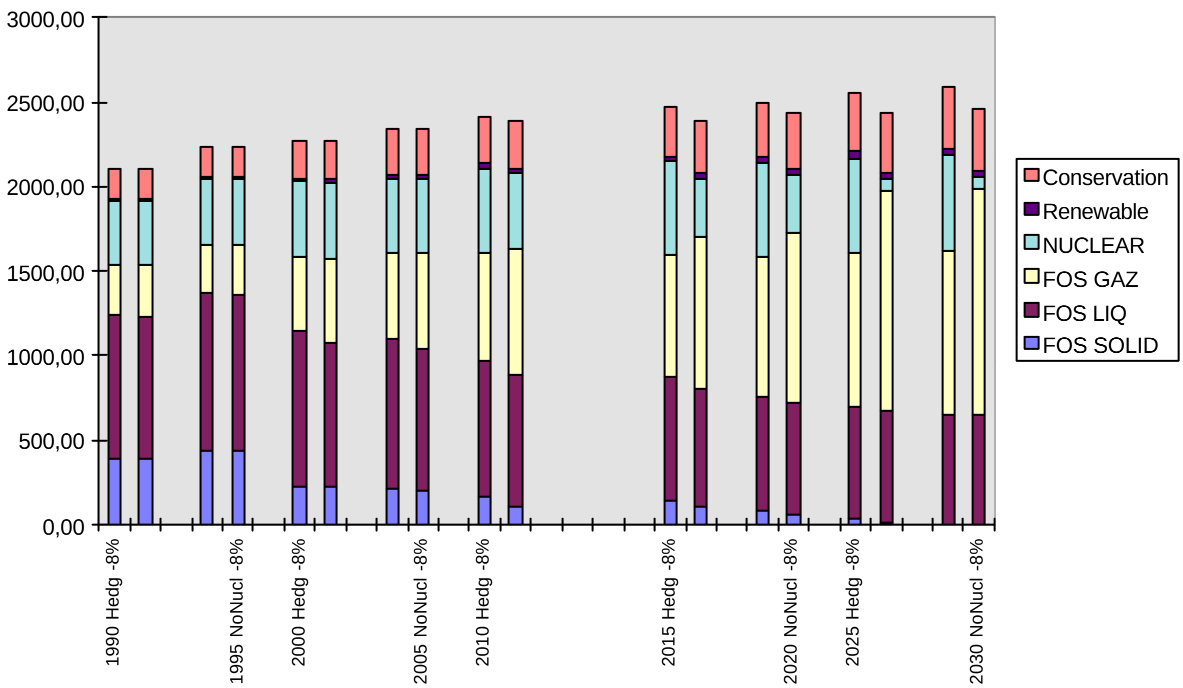
F. Impact of a nuclear moratorium

Because of the importance of nuclear power when CO2 constraints are imposed, a “NoNucl” scenario is considered, which is the same as the previous scenario with one exception: investments in new nuclear units are not allowed for. We will compare the stochastic solutions for both scenarios for the case that after 2012 it appears that the -8% cumulative constraint has to be satisfied.

The total discounted system cost for “NoNucl -8%” is 0,7% higher than for “Stoch. -8%”. Until the year 2005 in both scenarios the path for nuclear energy is fixed by assumption and therefore only after 2005 a difference appears. The use of nuclear energy is 10,5% lower under “NoNucl -8%” than under “Stoch. -8%”. In 2015 and 2020 it is 38% lower and in 2025 and 2030 it is 88% lower. There is also a decrease of the primary demand of solid fossil fuels. All this is compensated by a higher use of gas under “NoNucl -8%”.

Table 23 and Figure 20: Difference in primary input between No Nucl. -8% and Stoch. -8%

		1990	1995	2000	2005	2010	2015	2020	2025	2030
Stoch -8%	**FOS SOLID**	388,3	438,1	226,3	217,8	160,3	141,4	85,2	39,7	4,4
NoNucl -8%	**FOS SOLID**	390,5	438,3	226,5	202,0	107,3	99,4	53,6	14,5	4,4
	FOS SOLID	0,6	0,0	0,1	-7,3	-33,1	-29,7	-37,1	-63,5	0,0
Stoch -8%	**FOS LIQ**	850,1	931,6	917,8	884,3	808,1	729,4	667,2	662,0	647,5
NoNucl -8%	**FOS LIQ**	843,8	924,8	849,1	840,6	780,9	701,9	669,8	655,2	643,2
	FOS LIQ	-0,7	-0,7	-7,5	-4,9	-3,4	-3,8	0,4	-1,0	-0,7
Stoch -8%	**FOS GAZ**	300,6	284,4	438,7	501,3	641,2	723,6	833,5	911,2	973,7
NoNucl -8%	**FOS GAZ**	305,1	293,0	501,4	561,9	744,1	904,5	1006,3	1309,8	1343,0
	FOS GAZ	**1,5**	**3,0**	**14,3**	**12,1**	**16,0**	**25,0**	**20,7**	**43,7**	**37,9**
Stoch -8%	**NUCLEAR**	381,9	395,8	446,3	446,3	498,8	556,6	557,0	557,0	557,0
NoNucl -8%	**NUCLEAR**	381,9	395,8	446,3	446,3	446,3	345,0	345,0	64,4	64,4
	NUCLEAR	0,0	0,0	0,0	0,0	**-10,5**	**-38,0**	**-38,1**	**-88,4**	**-88,4**
Stoch -8%	**Renewable**	8,1	8,1	21,9	24,4	26,8	29,1	33,1	41,7	44,8
NoNucl -8%	**Renewable**	8,1	8,1	21,9	24,4	26,8	30,3	33,1	35,9	44,8
	Renew.	0,0	0,0	0,0	0,0	0,0	4,1	0,0	-13,9	0,0
Stoch -8%	**Conserv.**	182,2	176,0	223,8	261,9	275,3	290,5	317,7	341,7	359,1
NoNucl -8%	**Conserv.**	180,4	176,0	225,7	262,2	279,1	304,4	332,2	350,6	359,1
	Conserv.	-1,0	0,0	0,8	0,1	1,4	4,8	4,6	2,6	0,0



V. Conclusion

After a short description of the theoretical background for analysing investment decisions under uncertainty, an application with the Markal model is presented. The focus lies on the comparison of a stochastic strategy with the deterministic strategies.

The stochastic strategy has the advantage, to define "one" strategy which takes into account the possible constraints that can be imposed after a certain period. It allows to keep a certain flexibility before the uncertainty is resolved and this comes clearly out of the comparison.

This exercise will be repeated when the Belgium Markal database will be updated, because the flexibility of the energy system is overvalued, by the way some fuel switch technologies are now modelled in Markal.

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VII. Annexes

A. MARKAL Versions

MARKAL computes a competitive partial equilibrium on the energy market, where the endogenous energy prices are equal to the marginal costs of the energy vectors and where demands for energy services are exogenously set by scenario. We used a stochastic version of this model.

MARKAL-micro is an extension of the MARKAL-model. In this model the demands for energy services are elastic to their own prices. Instead of fixing demands, the user specifies demand functions. The elasticity may be different for different demand categories and for different time-periods. The model should prove useful in the context of analyzing scenarios where environmental taxes or constraints impose a non-negligible strain on the various economic sectors in the form of severe increases in the marginal cost of some energy. This model captures the greatest part of the feed-back effects not previously acounted for in MARKAL. It still is not a general equilibrium model, because there is no adjustment in the model for changes in the macro-economic variable for GDP (Denise Van Regemorter and Gary Goldstein).

MARKAL-ED is almost the same as Markal-micro, but in Markal-ED the objective function is made linear by using stepwise functions (Dennis Lavigne and Garry Goldstein).

MARKAL-macro goes a step further concerning the impact on the macro-economic variables (e.g. changes in GDP). Markal-macro is in this respect a general equilibrium model. However, because of the size, only one price elasticity is assumed for all sectors.

For more information take a look at the following address on the internet: http://www.ecn.nl/unit_bs/etsap/markal/ .

B. Risk aversion

1. Definition of a "risk averter" and a "risk neutral person"

A decision maker is a *risk averter*, if for any distribution F(x) the following is true:

receiving the amount $\int_{-\infty}^{+\infty} x\, dF(x)$ with certainty is considered at least as good as taking part in a lottery with distribution F(x). This can be represented by:

$u\left(\int_{-\infty}^{+\infty} x\, dF(x)\right) >= \int_{-\infty}^{+\infty} u(x)\, dF(x)$."

The above inequality is called Jensen's inequality and is the defining property of a concave function. Hence in the context of the expected utility theory, risk aversion is equivalent to the concavity of u(..).

For a risk neutral person we have: for all F(...): $u\left(\int x\, dF(x)\right) = \int u(x)\, dF(x)$

2. Measuring risk aversion

Representations of a preference ordering by utility functions are not unique (Spinneweyn F, 1989). The class of utility functions representing the same preference ordering is more restricted under uncertainty than under certainty. Under certainty, positive monotonic transformations of a utility function, do not alter the preference ordering. Under uncertainty only positive linear transformations represent the same ordering.

Risk aversion is determined by the form of the utility function. A possible measure of risk aversion therefore is $u''(x)$. However, because this measure is not invariant to positive linear transformations of the utility function, it has not become a standard measure of risk aversion.

In the economic literature *The Arrow-Pratt (A-P) coefficient* is the standard measure of risk aversion. The Arrow-Pratt coefficient of *absolute risk aversion* is the simplest modification of $u''(x)$ that is invariant for positive linear transformations. It is defined as: $-u''(x)/u'(x)$. The negative sign results in a positive Arrow-Pratt coefficient for a concave and increasing utility function. The more concave is the utility function, the higher will be the degree of risk aversion and the larger will be the A-P coefficient.

The A-P coefficient can be used to compare the risk attitudes of individuals with different utility functions. The A-P coefficient can be used as well to compare the risk attitude of one individual at different levels of wealth.

Instead of the A-P coefficient of absolute risk aversion, sometimes the A-P coefficient of *relative risk aversion* is used: $-x\, u''(x) / u'(x)$. The concept of relative risk aversion is particular interesting for analysing risky projects where outcomes are expressed in percentages of gains or losses of current wealth.

It is observed that an individual is more willing to take a risk, when he is rich than when he is poor. If somebody is rich, he can afford to take a risk. This behaviour can be modelled by assuming that the Arrow-Pratt coefficient of absolute risk aversion decreases when the wealth increases. If this is the case one speaks of *"decreasing absolute risk aversion"*

The assumption of decreasing absolute risk aversion yields many economically reasonable results concerning behaviour under risk. However this assumption may be too weak and may therefore be replaced with the stronger assumption of *"non-increasing relative risk aversion"*. This means that the Arrow-Pratt coefficient of relative risk aversion does not increase when the wealth level increases.