The relative efficiency of environmental policy instruments in a second-best setting with costly monitoring and enforcement[1]

Sandra ROUSSEAU and Stef PROOST

Centre for Economic Studies

K.U.Leuven

Naamsestraat 69

B-3000 Leuven

Belgium

E-mail: *Sandra.Rousseau@econ.kuleuven.ac.be*

Stef.Proost@econ.kuleuven.ac.be

Abstract

In this paper we incorporate monitoring and enforcement aspects in the choice of environmental policy instruments in a general equilibrium framework. Goulder et al. (J.Pub.Econ., 1999) look into the choice of policy instruments in the presence of distortionary taxes. We extend this model by no longer assuming full compliance from firms. A violating firm is caught with a certain probability by the inspection agency. Once a violator is detected, he always has to pay a fine. With a positive, finite expected fine and a probability of detection smaller than unity, there will always be a certain proportion of noncompliance in the economy. We calculate the gross efficiency costs of different policy instruments (emission tax, output tax, tradable permits and technology mandate). We illustrate the model for different price instruments (emission tax, output tax and tradable permits). We find that the relative inefficiency of grandfathered tradable permits vis-à-vis emission taxes found in a second-best setting with perfect compliance, is strongly decreased with imperfect compliance.

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1.   Introduction

When creating an environmental policy the regulator faces many choices. The most important one is probably the selection of the appropriate policy instrument. Many criteria are relevant for this choice. In the literature[2] one finds economic criteria (e.g. economic efficiency, cost-effectiveness[3] or distribution), environmental criteria (e.g. threshold safety levels, dose-response relationships or irreversibilities), technological criteria[4] (feasibility or incentives for innovation) and political criteria[5] (equity, precaution, acceptability and simplicity). Here we focus on economic efficiency and analyse the integration of monitoring and enforcement in traditional economic models for the selection of policy instruments.

In recent years, attention has been paid to the interaction of the environmental policy with distortionary taxes, such as labour taxes. It turns out that environmental taxes can increase the inefficiency of existing labour taxes. In fact it is shown that reducing pollution, no matter how, has mostly a hidden cost when there are existing tax distortions[6]. Goulder, Parry and Burtraw (1997) have used analytical and numerical general equilibrium models to explore the choice between pollution taxes and quotas in the presence of distortionary labour taxes.

This paper extends the work of Goulder et al. (1999). They use a general equilibrium model to examine the costs of achieving pollution reductions under a range of environmental policy instruments in a second-best setting with pre-existing labour taxes. They compare the costs and overall efficiency impact of emission taxes, emission quota, fuel taxes and mandated technologies. These efficiency costs are decomposed into four terms: the abatement effect, the output substitution effect, the revenue-recycling effect and the tax interaction effect. All these models use the assumption of perfect and costless monitoring and enforcement of the different environmental policies. This assumption is not unimportant. Although rigorous analyses of inspection and enforcement problems are rare, enforcement problems are often quoted as one of the major decision criteria in the choice of instruments. Enforcement problems may require important public budgets and may limit the environmental effectiveness of certain policy lines.

Monitoring and enforcement issues have been studied in the literature in a detailed way but mostly instrument by instrument. The goal of this paper is not to optimise the enforcement strategy for each instrument but rather to compare instruments given imperfect compliance. This will require strong simplifications on the behaviour of polluting firms and that of the inspection agency.

In this paper we choose to deter firms by the threat of high expected penalties. One of the first questions to ask is why firms would attempt to comply with regulations. At first sight they gain nothing by complying, on the contrary they face higher costs. However, several motivations for compliance can be found in the literature (e.g. Cohen – 2000). Firms can be deterred if they face high expected penalties (e.g. Becker – 1968). State dependent

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[2] See, for example, Bohm and Russell (1985).

[3] See, for example, Atkinson and Tietenberg (1991); Parry (1997).

[4] See, for example, Jaffe and Stavins (1995); Milliman and Prince (1989).

[5] See, for example, Hahn (1990).

[6] See, for example, Goulder (1995), Bovenberg and de Mooij (1994) and Parry (1995).

enforcement (e.g. Harrington – 1988, Harford – 1991, Harford and Harrington – 1991) can also induce firms to comply with environmental policy. In these papers firms are placed in a certain group depending on their past compliance behaviour. Firms who violated the law in the previous period face a higher probability of inspection than compliant firms do. In a recent paper Heyes and Rickman (1999) consider regulatory dealing as a possible cause of the compliance behaviour by firms. Another plausible explanation is that firm managers are led by social norms and accordingly adjust their behaviour (e.g. Arora and Gangopadhyay – 1995, Rauscher – 1997). Another way to obtain greater compliance is by incorporating self-reporting (e.g. Kaplow – 1995, Livernois and McKenna – 1999, Heyes – 1996).

There is some controversy in the literature concerning the objective function of the inspection agency. Some authors assume that the agency maximises environmental quality; e.g. Farber and Martin (1986). It is also possible that the inspection agency maximises social welfare; e.g. Polinsky and Shavell (1992). If this is the case the governmental and the agency's choice problems can be aggregated since they share the same objective function. This is the approach we will follow. Another option for the inspection agency is to maximise compliance or minimise violations; e.g. Garvie and Keeler (1994). Finally we also like to mention the minimisation of the enforcement costs as a regulatory objective; e.g. Storey and McCabe (1980), Malik (1992). Which objective function to choose is a matter of individual preference. Keeler (1995) compares the consequences of different objective functions. He concludes that greater weight given to compliance costs relative to the social damages of the polluting activity will bring outcomes closer to the optimum when the regulator is strong, but may move the outcome further from the social optimum when regulatory powers are weak.

A noncompliant firm will be detected with a certain probability. We assume that an inspection team can perfectly detect the actual level of emissions. Consequently all violators are caught if they are inspected[7]. We will assume one particularly easy form for the probability of detection: it is proportional to the percentage underreporting. However, these inspections are not for free. We assume that inspection costs are financed by the government budget.

Once the violator has been caught, the firm has to pay a fine, consisting of the overdue taxes and a penalty payment.

In section 2 monitoring and enforcement issues are integrated into the Goulder et al. model. In section 3 we compare the gross efficiency costs of marginal policy changes with an analytical model. In section 4 we describe the numerical model. This model is used to calculate the relative cost efficiency of different policy instruments for several levels of emission reduction in section 5. In section 6 we discuss the role of the fine and the inspection costs extensions and in section 7 we conclude.

2.   The model

The model we use is based on the model of Goulder et al. (1999). We discuss the representation of household behaviour, firms and government.

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[7] Papers that relax this assumption are Linder and McBride (1984), Heyes (1994), Beavis and Walker (1983), Bose (1995) and Harford (1991).

2.1   Households

It is a static, general equilibrium model where a representative household derives utility from the consumption of a polluting good *(X)*, a clean good *(Y)*; and from leisure. Leisure is equal to the household time endowment *(T)* less labour supply *(L)*. The emissions *(E)* resulting from the production of *X* cause environmental damage and therefore lead to a decrease in consumer utility. The household utility function is:

$$U = u(X, Y, T - L) - f(E) \qquad (1)$$

where *u*(.) is a quasi-concave utility function for non-environmental goods. It is assumed that X and Y are equally good substitutes for labour. The function f(.) is the disutility from waste emissions. It is weakly convex. The separability restriction in (1) implies that demand for *X* and *Y* and supply of labour do not vary with changes in E[8]. Therefore we can leave the the improvement in environmental quality out of the picture. In this paper we discuss the gross efficiency costs of decreasing emissions by a given level. We will only focus on efficiency considerations and will ignore distributional aspects.

The household budget constraint is:

$$p_X X + Y = (1 - t_L)L + G + p \qquad (2)$$

where p_X is the demand price of *X*. This price is equal to unity in the absence of regulation. The price of the non-polluting good p_Y is constant, equal to unity and not affected by the environmental policy. The non-polluting good *Y* is the numeraire in the theoretical model. In the numerical model we will use labour as the numeraire. The firms' profits π are redistributed to the households.

The households choose *X*, *Y* and *L* in order to maximise utility (1) subject to the budget constraint (2), taking environmental damages as given. From the resulting first-order conditions the uncompensated demand and labour supply functions are obtained:

$$X(p_X, 1 - t_L, G + p), \quad Y(p_X, 1 - t_L, G + p) \text{ and } L(p_X, 1 - t_L, G + p) \qquad (3)$$

Substituting these equations into (1) gives the indirect utility function:

$$V = v(p_X, 1 - t_L, G + p) - f(E) \qquad (4)$$

2.2   Firms

Competitive firms use labour, which is the only factor of production, to produce the goods *X* and *Y*. The marginal product of labour is assumed to be constant in each industry. Further output is normalised such that marginal products and wage rate equal unity.

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[8] For a model without this separability assumption see Mayeres and Proost (1997).

Firms can reduce their waste emissions per unit of output by using abatement equipment or purchasing abatement services. It is assumed that such equipment or services are produced directly from labour. Emissions per unit of X are equal to e_o-a, where e_o represents baseline emissions per unit of output (or emissions per unit without regulation) and a is the reduction in per-unit emissions due to abatement. Economy-wide emissions, E, are therefore equal to $(e_o-a)X$. Thus, total emissions fall as a result of reduced production of X (the output-substitution effect) and increased abatement activity (the abatement effect). In the numerical model we will incorporate a third way of emission reduction, namely via input-substitution. This means that firms will have the option to change their input mix and use less of the most polluting input.

The total cost C of abatement to the firm is given by:

$$C = c(a).X \tag{5}$$

where $c(a)$ is a convex function representing the per-unit cost of abatement activity.

In order to incorporate monitoring and enforcement we need some additional assumptions.

As explained in the introduction we assume that firms will attempt to comply with the environmental regulation because of the threat of high expected penalties. Furthermore we also assume that firms can be noncompliant in a continuous way. They can choose any possible level of actual emissions. When we consider an emission tax (t_E), firms have to report their emissions (Z_j) to the government. It is obvious that reported emissions will never exceed actual emissions if firms behave rationally. The regulator, however, anticipates that firms are inclined to cheat and will therefore pursue a monitoring and enforcement policy in order to deter them.

The noncompliant firm will be detected with a certain probability, the probability of detection p_{det}. We assume that the equipment of an inspection team perfectly measures the actual level of emissions. Consequently all violators are caught if they are inspected. We will assume that the probability of detection is proportional to the underreporting[9]. Inspections are costly. Firstly we have fixed inspection costs (FIX), e.g. infrastructure. The level of these costs does not depend on the number of inspections performed, only on the type of policy instrument used. Further we also assume that there are variable inspection costs (v), e.g. sample testing or fuel. These costs are incurred every time an inspection is performed. All these costs are financed by the government budget.

Once the violator is caught, the firm has to pay the overdue taxes plus a penalty (r) payment that is proportional to the evaded taxes. This total payment is called the fine (f) and is equal to $f = (e - z) \ t_E (1 + r)$.

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[9] The inspection agency will inspect firms with a higher level of violation with a higher probability. We assume that the greater the crime the more visible it is for the inspection agency. This higher visibility can be the result of complaints by neighbours and interest groups or it can result from higher nuisance. The agency does not take any other attributes of firms into account.

2.3   The behaviour of the firms

Let us consider a revenue-neutral tax t_E on the emissions in the industry. The profit for the firm per unit of X is:

$$p_X - \left\{ \ 1 + c(a) + t_E.z + p_{\det}.f \right\} \tag{6}$$

with

$$p_{\det}.f = \left(\frac{e_0 - a - z}{e_0 - a} \right).(e_0 - a - z)t_E(1+r)^{10} \tag{7}$$

z = reported emissions per unit of output

r = interest payments on overdue taxes

and, since we work in competitive markets, profits are zero in equilibrium .

Note that for $p_{\det} = 0$ or $f = 0$, we get a corner solution and the reported emissions will equal zero. Since violators are not punished for lying about their emissions, they maximise their profits by reporting no emissions and therefore paying no taxes.

In table 2 we investigate the influence of the emission tax t_E and the penalty[11] r on the optimal behaviour of firms. More specifically we look at changes in abatement a^* and reported emissions z^*.

t_E	r	z^*	a^*
$= 0$	≥ 0	$z = 0$	$a = 0$
> 0	$= 0$	$z = \frac{1}{2}(e_0 - a)$	$a = \frac{3}{4}t_E$
> 0	$\to \infty$	$z \to (e_0 - a)$	$c'(a) \to t_E$
> 0	> 0	A. $z = (e_0 - a)$	A. $c'(a) = t_E$
		B. $z = \left(\frac{1+2r}{2+2r} \right)(e_0 - a)$	B. $c'(a) = t_E \left(\frac{3+4r}{4+4r} \right)$

Table 1: Reaction of firms to changes in the emission tax and the penalty

Firstly we consider the case were the emission tax is zero. Firms will not invest in abatement nor will they report any emissions. Therefore we will focus on a strictly positive emission tax from now on.

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[10] The government can only observe actual emissions after inspection of the firm. However, they do assume that firms act as if $p_{\det} = \dfrac{e - z}{e}$. This is plausible since inspections often follow complaints by citizens or other administrations and these complaints are more probable if violations are more serious.

[11] Please keep in mind that the penalty r is not equivalent to the fine f.

Secondly for r = 0 the interest payments are zero but there is still a punishment for violating firms because they have to pay their overdue taxes. The reported emissions will not be zero because the reported emissions influence the probability of detection. The total amount paid, taxes on reported emissions plus overdue taxes, is minimised by reporting half of the actual emissions.

Thirdly when the penalty r goes to infinity, the firms will be reporting more and more truthfully. Moreover, the marginal abatement cost will, in the limit, equal the emission tax rate.

Finally we consider the case in which both the emission tax and the penalty are positive and finite. The firms will never report zero emissions because then the firm would always have to pay the complete tax plus the fine. It could always do better by reporting truthfully because then it would not have to pay the fine. We are now left with two cases: one is the corner solution of reporting truthfully and one leads to an internal solution. Reporting truthfully (case A) will only be optimal when the following expression is satisfied:

$$p_X - \left\{1 + c(a) + t_E.(e_0 - a)\right\}\Big|_{c'(a)=t_E} > p_X - \left\{1 + c(a) + t_E.(e_0 - a)\frac{3+4r}{4+4r}\right\}\Big|_{c'(a)=t_E\frac{3+4r}{4+4r}} \qquad (8)$$

We have evaluated this expression for a linear abatement cost function and it never holds. For different specifications of the abatement function, e.g. a quadratic function, the situation will probably differ. However we choose to ignore this case. We will focus in this paper on the existence of an interior solution (Case B).

In order to obtain an expression for the reported emissions per unit output (z), we differentiate equation (6) with respect to z. This gives:

$$er = \left(1 - \frac{1}{2(1+r)}\right)(e_0 - a) = \left(\frac{1+2r}{2+2r}\right)(e_0 - a) \qquad for f, p_{det} \neq 0 \qquad (9)$$

We now substitute (7) and   (9) into equation (6) and obtain that the profit per unit of output is equal to:

$$p_X - \left\{1 + c(a) + t_E.(e_0 - a)\frac{3+4r}{4+4r}\right\} \qquad (10)$$

Firms choose a, the level of abatement per unit, in order to maximise profits (differentiate (10) with respect to a). The first-order conditions are:

$$t_E\left(\frac{3+4r}{4+4r}\right) = c'(a) \qquad (11)$$

For each firm abatement activity occurs until the marginal abatement cost per unit of X equals the effective emission tax rate.

The government levies a proportional tax of t_L on labour earnings, regulates emissions and provides a fixed nominal lump-sum transfer G to the households. The government budget is

assumed to be balanced. Adjusting the labour tax t_L [12] offsets any revenue consequences from environmental policies. In the numerical model we will assume that rent income is taxed at the same rate as labour income.

2.4   Government

The expression for the government budget is:

$$t_E Z + t_L L + p_{\det}.f.X = G + FIX + v.p_{\det}.n \qquad (12)$$

where n represents the number of firms, v is the variable inspection cost and FIX represents the fixed cost of the inspection. In principle inspection cost functions can differ for each environmental policy instrument.

3.   The gross efficiency cost of different environmental instruments

Goulder et al. focus on the gross efficiency cost of alternative environmental policies. This cost is the monetary equivalent of the loss in utility. It is a gross concept in that it does not include the environmental-related impacts on indirect utility from changes in $\boldsymbol{f}(.)$.

They use:

$$M \equiv \frac{-t_L \dfrac{\partial L}{\partial t_L}}{L + t_L \dfrac{\partial L}{\partial t_L}} \qquad (13)$$

This is the (partial equilibrium) efficiency cost from raising an additional dollar of labour tax revenue[13]. The numerator is the efficiency loss from an incremental increase in t_L. This equals the wedge between the gross wage (equal to the value of the marginal product of labour) and the net wage (equal to the marginal social cost of labour in terms of foregone leisure), multiplied by the reduction in labour supply. The denominator is the marginal labour tax revenue (from differentiating $t_L L$).

We now analyse the gross efficiency cost of various environmental policies for the case with imperfect compliance and have Goulder et al. results as a special case.

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[12] In a one-consumer setting, the optimal tax structure is to use only the lump-sum tax G and to have $t_L = 0$ or to use a profit tax in the presence of pure profit. In this case the Goulder et al. problem becomes trivial because we can return to a first-best if Pigouvian taxes can be used (Mayeres and Proost (1997)). However, this simple framework will allow us to better isolate the effects of the environmental policy.

[13] It equals the marginal cost of public funds minus one.

3.1   The gross cost of emission taxes

Next we consider an incremental, revenue-neutral increase in t_E. The effect on the product price of this increase is (differentiating (10) with respect to t_E):

$$\frac{dp_X}{dt_E} = (e_o - a)\left(\frac{3+4r}{4+4r}\right) \tag{14}$$

Revenues from the emission tax will be employed to finance cuts in the distortionary tax, t_L. We now derive an expression for the change in labour tax necessary to maintain government budget balance following the increase in emission tax.

Totally differentiating the government budget gives (holding G constant)[14]:

$$\left(\frac{3+4r}{4+4r}\right)\left(E + t_E \frac{dE}{dt_E}\right) + \frac{dt_L}{dt_E}L + t_L \frac{\partial L}{\partial p_X}\frac{dp_X}{dt_E} + t_L \frac{\partial L}{\partial t_L}\frac{dt_L}{dt_E} = 0 \tag{15}$$

$$\text{or } \frac{dt_L}{dt_E} = -\frac{\left(\dfrac{3+4r}{4+4r}\right)\left(E + t_E \dfrac{dE}{dt_E}\right) + t_L \dfrac{\partial L}{\partial p_X}\dfrac{dp_X}{dt_E}}{L + t_L \dfrac{\partial L}{\partial t_L}} \tag{16}$$

$$\text{with } \frac{dE}{dt_E} = \frac{\partial E}{\partial t_E} + \frac{\partial E}{\partial t_L}\frac{dt_L}{dt_E} \tag{17}$$

Using   (13), see appendix 1 (part 1), we obtain:

$$\frac{dt_L}{dt_E} = -\frac{(1+M)}{L}\left\{\left(\frac{3+4r}{4+4r}\right)\left(E + t_E \frac{dE}{dt_E}\right) + t_L \frac{\partial L}{\partial p_X}\frac{dp_X}{dt_E}\right\} \tag{18}$$

We are interested in the gross effect of the tax increase t_E on welfare. Differentiating utility (4) with respect to t_E, using Roy's identity and (14), and ignoring the terms in ϕ[15], gives (see appendix 1-part 2):

$$\frac{dv}{dt_E} = \frac{\partial v}{\partial p_X}\frac{dp_X}{dt_E} + \frac{\partial v}{\partial t_L}\frac{dt_L}{dt_E}$$

$$or \; -\frac{1}{\mathbf{1}}\frac{dv}{dt_E} = E + L\frac{dt_L}{dt_E} \tag{19}$$

This is the efficiency cost (ignoring environmental benefits) from an incremental increase in t_E, expressed in monetary terms.

Substituting (18) in (19) gives:

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[14] The monitoring and enforcement expenditures are constant due to the specific functional forms we chose.

[15] Remember that we look for the gross efficiency costs and therefore do not take the environmental effects of a policy into account.

$$-\frac{1}{I}\frac{dv}{dt_E} = t_E\left(-\frac{dE}{dt_E}\right)\left(\frac{3+4r}{4+4r}\right) - M\left(\frac{3+4r}{4+4r}\right)\left(E + t_E\frac{dE}{dt_E}\right) + (1+M)\left(t_L\left(-\frac{\partial L}{\partial p_X}\right)\frac{dp_X}{dt_E}\right)$$ (20)

Finally, using $E = (e_0 - a)X$ and (17), we obtain:

$$\frac{dE}{dt_E} = -\frac{da}{dt_E}X + (e_o - a)\frac{dX}{dt_E}$$ (21)

Substituting (21) in (20), using (14), gives us:

$$-\frac{1}{I}\frac{dv}{dt_E} = c'(a)\left(\frac{da}{dt_E}\right)\left(\frac{3+4r}{4+4r}\right)X + \left(-\frac{dX}{dt_E}\right)\left(\frac{3+4r}{4+4r}\right)t_E(e_o - a)$$

abatement   output-substitution

$$-M\left(\frac{3+4r}{4+4r}\right)\left(E + t_E\frac{dE}{dt_E}\right) + (1+M)\left(t_L\left(-\frac{\partial L}{\partial p_X}\right)(e_o - a)\left(\frac{3+4r}{4+4r}\right)\right)$$ (22)

revenue-recycling   tax interaction

$$\text{with}\quad \frac{dX}{dt_E} = \frac{\partial X}{\partial p_X}\frac{dp_X}{dt_E} + \frac{\partial X}{\partial t_L}\frac{dt_L}{dt_E}$$ (23)

As in Goulder et al. we find that the gross cost of an increase in the emission tax recycled via a decrease in labour taxes can be decomposed into four effects. The reduction in emissions is achieved via a combination of two effects: the reduction of emissions per unit of output (abatement effect) and the substitution away from the consumption of X (output-substitution effect). The costs caused by these two effects are called the primary costs. In a first-best setting, the relative cost-effectiveness of different policies can be explained fully in terms of differences in primary costs.

In a second-best setting, with distortionary taxes, two additional cost terms come into play. The first term is the efficiency gain from the (marginal) revenue-recycling effect. This is the product of the marginal excess burden of taxation and the marginal revenues (if any) from the policy. It represents efficiency gains associated with using these revenues to finance cuts in distortionary taxes.

The second extra term is the efficiency loss from the tax interaction effect. This effect has two components. First, the new policy can increase the price of X, implying an increase in the cost of consumption and thus a reduction in the real wage. This reduces labour supply and produces a marginal efficiency loss that equals the tax wedge between the gross and the net wage multiplied by the reduction in labour supply. In addition, the reduction in labour supply contributes to a reduction in tax revenues, which has an efficiency cost of M times the lost tax revenues, equal to the change in labour supply times the labour tax rate.[16]

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[16] Here we rule out complementarity between leisure and X.

We see that incorporating monitoring and enforcement in the model, leads to a decrease of the effective tax rate. The effective tax rate is equal to the nominal tax rate (t_E) times the factor $\left(\dfrac{3+4r}{4+4r}\right)$. This last factor depends on the penalty rate on overdue taxes. The effective tax rate is lower than the nominal one in a model with monitoring and enforcement because some firms can get away with only paying taxes on their reported emissions and not their actual emissions. Since the effective tax rate is lower in the model with monitoring and enforcement than in the model without, all the different efficiency effects will also be lower. Therefore we can say that the gross efficiency cost of a marginal increase in the nominal emission tax is lower in the model with monitoring and enforcement. Moreover the monitoring and enforcement costs do not differ at the margin.

However, all this does not mean that utility is higher for the model with partial compliance. Remember that we did not take the environmental effects of the policy into account. It is obvious that the environmental quality is worse for a particular tax rate in the partial compliance case. In order to reach the same environmental emission reduction as with full compliance, one needs a comparatively higher emission tax rate.

Compared to the Goulder et al. case, we obtain the same four effects all deflated with the same effective tax factor $\dfrac{3+4r}{4+4r}$. When r approaches infinity, there is full compliance and this factor disappears.

3.2   The gross efficiency cost of tradable permits

The derivations for the remaining instruments provide no new insights and we will therefore not elaborate on them.

It is assumed that a firm receives a certain number of permits from the government. It is possible to obtain more permits by trading among firms. Firms state the number (pr) of emission permits they own. If their actual emissions are fully covered by the permits a firm is in compliance. If the actual emissions exceed the number accounted for by the permits, the violator risks to be detected with a certain probability rate $p_{\text{det}} = \left(\dfrac{e_0 - a - pr}{e_0 - a}\right)$ and then they have to pay a fine equal to $f = (e_0 - a - pr)\, t_E^v (1 + r)$. The virtual tax can be interpreted as the price at which the permits are traded in the market[17].

For a tradable permit system, we analyse the gross tax of an emission permit reduction via an increase in the virtual tax t_E^v :

$$-\frac{1}{I}\frac{dv}{dt_E^v} = \left(\frac{3+4r}{4+4r}\right)t_E^v\left(\frac{da}{dt_E^v}\right)X + \left(\frac{3+4r}{4+4r}\right)\left(-\frac{dX}{dt_E^v}\right)t_E^v\left(e_0 - a\right)$$

$$\qquad\qquad\qquad\text{abatement effect}\qquad\qquad\qquad\text{output-substitution effect}$$

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[17] We assume that the trading of emission permits takes place in a perfectly competitive market.

$$-\frac{1}{4+4r}M\left(E+\ t_E^v\frac{dE}{d\ t_E^v}\right)+(1+M)t_L\left(-\frac{\partial L}{\partial p_X}\right)(e_0-a)\left(\frac{3+4r}{4+4r}\right) \qquad (24)$$

<div align="center">revenue-recycling   tax interaction effect</div>

Compared to the emission tax case, we have the same abatement, output-substitution and tax-interaction effect. The revenue-recycling effect, however, will be much smaller. This is in contrast with Goulder et al. results where, because of the full compliance assumption, there is no revenue-recycling effect for tradable permits. We will see later in the numerical results that this reduces the disadvantage of tradable permits in second-best worlds.

3.3   The gross cost of a fuel tax

For a revenue-neutral output tax t_X, the gross efficiency cost of an increase in t_X is equal to:

$$-\frac{1}{I}\frac{dv}{dt_X}=\left(\frac{3+4r}{4+4r}\right)\left(-\frac{dX}{dt_X}\right)t_X-M\left(\frac{3+4r}{4+4r}\right)\left(X+t_X\frac{dX}{dt_X}\right)$$

<div align="center">output-substitution effect   revenue-recycling effect</div>

$$+(1+M)t_L\left(-\frac{\partial L}{\partial p_X}\right)\frac{3+4r}{4+4r} \qquad (25)$$

<div align="center">tax interaction effect</div>

This can be interpreted in the same way as before. Again the effects are decreased by the factor $\frac{3+4r}{4+4r}$. The fine and the probability of detection depend on the difference between reported (Xr) and actual production (X).

3.4   The gross cost of a technology mandate

For the calculation of the gross efficiency cost of an incremental increase in a technology mandate $\overline{a}$ we work with slightly different assumptions. When we consider the expression of the firm's expected penalty, we get:

$$p_{det}.f=p_{det}.(\overline{a}-a)(1+r) \qquad (26)$$

We now assume that the probability of detection is fixed and independent of the true emissions. When a violator is caught, he has to pay a certain sum of money depending on the amount of abatement he failed to install.

For a revenue-neutral increase in the technology mandate, the gross efficiency cost is now equal to:

$$-\frac{1}{I}\frac{dv}{d\overline{a}}=-\ p_{det}(1+r)\frac{da}{d\overline{a}}X\ +p_{det}(1+r)\ (\overline{a}-a)\frac{dX}{d\overline{a}}$$

<div align="center">abatement   output substitution</div>

$$-M \quad p_{\det}\left(1+r\right)\left(\left(\overline{a}-a\right)\frac{dX}{d\overline{a}}+X\left(1-\frac{da}{d\overline{a}}\right)\right)+\left(1+M\right)\; t_L\left(-\frac{\partial L}{\partial p_X}\right)p_{\det}\left(1+r\right) \qquad (27)$$

<center>revenue recycling   tax interaction</center>

This can interpreted in the same way as before. We see that every term is multiplied by the same term $p_{det}(1+r)$. Moreover we also find an output substitution and a revenue recycling effect. This is due to the existence of fine payments to the government.

4. Numerical model

4.1 Description of the model

We use the same model and data as Goulder et al.. It is a general equilibrium model for the American economy, calibrated tot the 1990 situation. The environmental pollution problem addressed is the air pollutant NO_X.

We now incorporate intermediate inputs in the production model. This yields a new channel for emission reduction: the input-substitution effect. Emissions can be reduced not only by abatement and output-substitution but also by input-substitution. This means that the firms can alter the mix of intermediate inputs and use less of the polluting input.
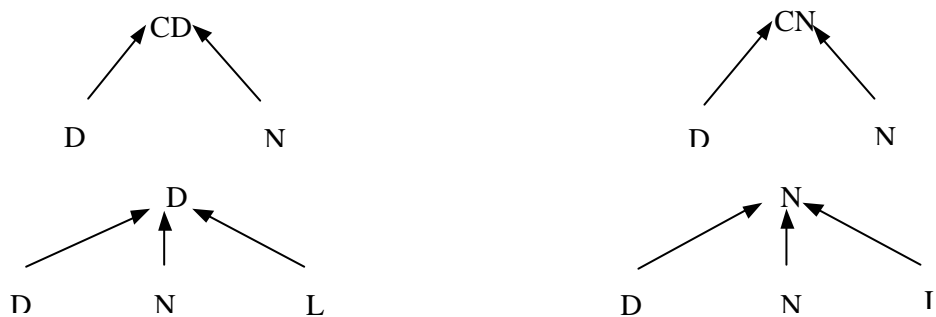

Figure 1: The production of the different commodities

We distinguish two different intermediate goods: a polluting (D) and a clean (N) intermediate good. In our application the polluting good can be thought of as being energy. There are two final consumption goods: CD represents output from industries that use D more intensively and CN represents output from industries that use N more intensively. The production relationships between different commodities are represented in Figure 1.

The structure of the numerical model is directly based on the previously discussed theoretical model. Household and firm behaviour, as well as governmental policy, are formalised. Labour is our numeraire. The complete model can be found in the appendix 2.

We assume that the representative household has a nested constant-elasticity-of-substitution (CES[18]) utility function:

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[18] Further reading on CES functions can be found in Keller (1976).

$$U = U\left(l,\ CD,\ CN,\ E\right) = \left(\boldsymbol{a}_{leis}\ leis^{\frac{s_u-1}{s_u}} + \boldsymbol{a}_C\ C^{\frac{s_u-1}{s_u}}\right)^{\frac{s_u}{s_u-1}} \tag{28}$$

where C is composite consumption and defined by:

$$C = \boldsymbol{h}\left(\boldsymbol{a}_{CD}\ CD^{\frac{s_c-1}{s_c}} + \boldsymbol{a}_{CN}\ CN^{\frac{s_c-1}{s_c}}\right)^{\frac{s_c}{s_c-1}} \tag{29}$$

The definitions of the variables and parameters can be found in appendix 2. The household maximises utility with respect to the budget constraint:

$$p_{CD}CD + p_{CN}CN = p_L(1-t_L)L + \boldsymbol{p}(1-t_R) + p_C\ G \tag{30}$$

This expression equals household spending with the household revenues.

We now consider the production side of the economy. A CES-form is used for the production functions in all industries j:

$$X_j = \boldsymbol{d}\left(\sum_i \boldsymbol{a}_{ij} X_{ij}^{\frac{s_j-1}{s_j}}\right)^{\frac{s_j}{s_j-1}} \tag{31}$$

We assume a constant-returns-to-scale production technology. Labour and rent income are assumed to be taxed at the same rate[19]. We also assume that the marginal abatement costs are linear.

Firms choose profit-maximising production and abatement subject to the constraints imposed by environmental regulation and taking input and output prices as given. Profits equal the value of output minus expenditures on inputs, labour and abatement, less any tax and fine payments. This gives the following expression:

$$\boldsymbol{p}_j = (p_j - \boldsymbol{t}_j)X_j - \sum_i p_i X_{ij} - \boldsymbol{t}_e Z_j - A_j - p_{\det j}.F_j \tag{32}$$

To obtain a general equilibrium, supply must equal demand for all produced goods, government revenue must equal government transfer payment, and pollution emissions must equal a specified target. Since production and abatement functions are linearly homogeneous, the supply of each good is perfectly elastic at given factor prices and tax rates. Under these conditions we can reduce the set of equilibrium conditions to three equations:

aggregate labour demand equals aggregate supply: $L = AD_L + \sum_j A_j$,

government revenue equals expenditures: $REV = p_C G + FIX + \sum_j v.p_{\det j}$

and pollution levels equal the target level: $Etot = (1 - reduc)Etotmax$ [20].

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[19] In the numerical exercise both tax rates are reduced if we have extra revenue from the environmental policy. It would also be possible to adapt only the labour tax rate. This would not change the results for the instruments with zero profit for the firms. For the other instruments we expect some small changes.

4.2   Data

In Table 2 we summarise the benchmark data set of Goulder et al. (1998), which represents the United States economy in 1990. Production data were obtained from the Commerce Department Bureau of Economic Analysis.

The polluting intermediate good D comprises fossil fuels[21], while the clean intermediate good N includes all other intermediates. The final good CD is a composite of the consumer goods whose production involves intensive use of fossil fuels (consumer utilities, motor vehicles and gasoline), while the good CN embraces all other final goods.

The parameter values used in the model can be found in the appendix 2. The distribution parameters α for production and utility functions were calibrated in GAMS based on the assumed elasticities of substitution and the restriction that the benchmark data must be replicated in the absence of a new environmental policy.

	D	N	CD	CN	Leisure time	Total output value
D	91 441.0	111 842.7	156 881.1	6 264.4		366 429.2
N	88 073.5	4 741 097.5	464 159.9	2 670 485.6		7 963 816.5
Labour	186 914.7	3 110 876.3			1 832 106.1	5 129 897.1
Total output value	366 429.2	7 963 616.5	621 041.0	2 676 750.0		
Emissions (millions of KGs)	23 000					

Table 2: Input-output flows (in millions of 1990 dollars per year except when otherwise noted)

Source: Goulder et al. (1998)

Although we try to derive general relationships, we must commit to certain parameter values in running the model. The central case values for pollution-related parameters are based on characteristics of NO_x emissions (Pechan – 1996). Pollution takes place every time a unit of D is used in the production process.

In de numerical model we arbitrarily choose the values of the monitoring and enforcement costs. This exercise is illustrative of the changes that occur in a model when these costs are incorporated. Accurate data on these costs are very hard to find.

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[20] In the GAMS model we only use the government budget and emission target conditions. By Walras' law, if these two conditions are satisfied, the third condition must also hold. We have used this third condition, labour market clearance, as a check on the optimality of the obtained solution.

[21] Fossil fuels include oil, coal and natural gas.

4.3   Instruments

We consider three different environmental instruments: an emission tax, a fuel tax and tradable permits. We limit ourselves to price instruments[22] in the numerical model.

Emission tax

We will determine one emission tax rate t_e for all industries. Since in equilibrium the marginal abatement costs will be the same across industries, this assumption is imposed in the model. This is done via the first-order condition of the profit maximisation problem of firms. Firms will initially pay taxes on reported emissions. With a certain probability of detection, depending on the level of the violation, firms are caught. Then violators have to pay the overdue taxes increased with interest payments.

Fuel tax

The fuel tax is a tax t_D on the output of sector D; D being the polluting intermediate good. Firms will never use abatement to reduce their emissions because the fuel tax is levied on the amount of output of sector D and this is independent of the abatement expenditures by firms. This implies that marginal abatement costs in the different industries will equal zero. Firms will initially pay taxes on reported output. With a certain probability of detection, depending on the level of the violation, firms are caught. Then violators have to pay the overdue taxes increased with penalty payments.

Tradable permits

As mentioned before, we only discuss grandfathered tradable permits. In the model we look at the situation after trade has occurred. We assume that there are no transaction costs in the trading market. The model then determines the level of permits (or emissions[23]) per sector for which the marginal abatement costs are equal. Firms will initially claim that the emissions produced equal the number of permits owned. However, this is not necessarily true. With a certain probability of detection, depending on the level of the violation, firms are caught lying. Violators then have to buy extra permits and have to pay a certain fine, again depending on the size of the violation.

5.   Comparing the gross cost of different policy instruments

We will first determine the impact of a policy. Using initial prices we define the welfare gain as that sum of money which the households would have accepted in the initial position as equivalent to the impact of the reform, and we call this the equivalent gain (King – 1983). It is defined by[24]:

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22 However we are currently extending the model to quantity instruments, such as technology mandates, performance standards and emission standards.

[23] We assumed that one permit allows a firm to produce one unit (=kg) of emissions.

[24] with E(.) = expenditure function, R = the reference value and N = the new value

$$EG = equivalent\,gain = E\left(p^R, U^N\right) - E\left(p^R, U^R\right) \qquad (33)$$

In the reference situation we assume that there are no emission reductions and no environmental policy. The elements of the reference price vector p^R are all assumed to be equal to unity. Therefore we can rewrite the equivalent gain, using $E\left(p^R, U^N\right) = \dfrac{U^N}{p_u}$, as follows[25]:

$$EG = U^N - U^R. \qquad (34)$$

In each new scenario we impose a certain level of emission reduction, e.g. 10 percent. For every policy instrument we then determine the optimal size so that the required emission reduction is achieved. With each scenario a new utility level is associated. It is obvious that after the introduction of the environmental regulation utility will have been reduced. This holds because we ignore any utility effect of improved environmental quality (i.e. leaving out the benefit of the policy, which gives us the gross cost, rather than the net welfare change). Therefore we will often speak of "equivalent loss" rather than "equivalent gain".

To facilitate comparisons across instruments we take the emission tax as a reference point. We will compare the equivalent loss of the instruments with that of the emission tax in each scenario. Remember that we consider qualitative, rather than quantitative differences across policies.

5.1   First-best setting: gross costs with and without perfect compliance

We first consider the equivalent gain in a first-best setting[26] ($t_L = 0$). Now only the primary costs will apply. The losses (or costs) under the different policy instruments are shown in Figure 2. The differences across policies are expressed as the ratio of total losses of the policy in question to total costs under the emission gain. Consequently the curve for the emission tax is constant at unity.

The curve for the tradable permits is equal to unity and therefore coincides with the curve for the emission tax. This means that, under a first-best setting, grandfathered tradable permits are just as efficient as an emission tax. This holds because the tax interaction and revenue-recycling effects do not prevail in the absence of distortionary taxes and thus the source of the cost differences, the revenue-recycling effect, is absent.

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[25] This holds only for the specific functional forms we use in our model.

[26] Any government revenues are returned in a lump-sum fashion to the households.
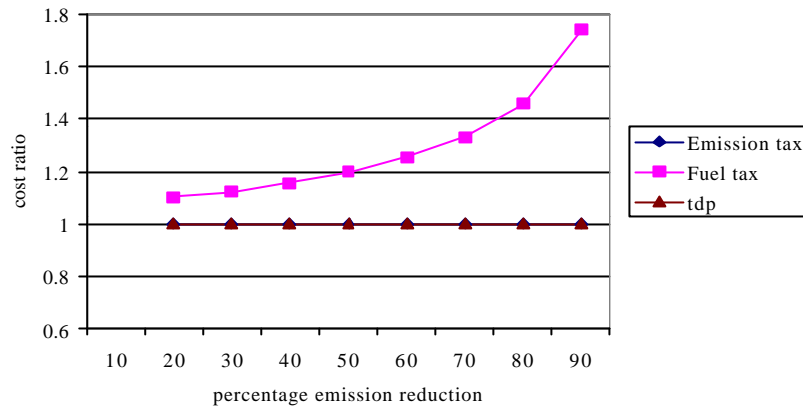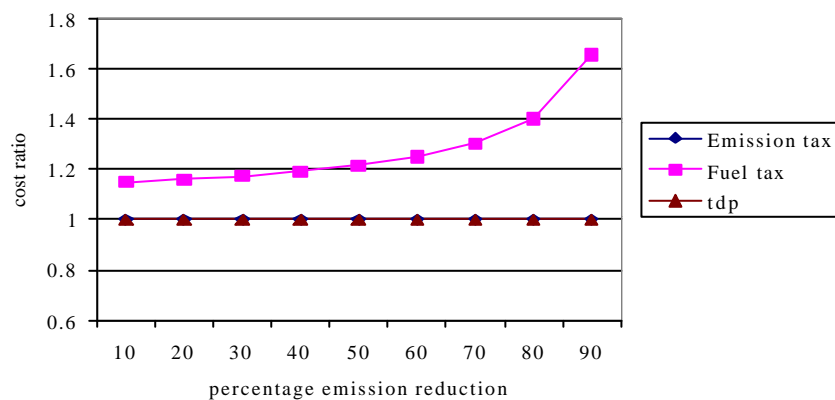
Figure 2: The cost ratio of first-best policy alternatives to the cost of a first-best emission tax

Next we consider the fuel tax. Its first-best cost exceeds that of the emission tax because the abatement effect is absent. Firms will not reduce emissions by installing abatement equipment because it does not help them to comply with the policy and it is costly.

We now compare these results with the results obtained in a model without monitoring and enforcement (see Figure 3).

Comparing the two figures, one can see that in a first-best setting the introduction of a monitoring and enforcement policy does not greatly alter the relative position of the different environmental policy instruments. Just note that, under full-compliance, the relative cost difference between the fuel tax and the emission tax is larger (smaller) for small (high) emission reductions than in the model with imperfect compliance.
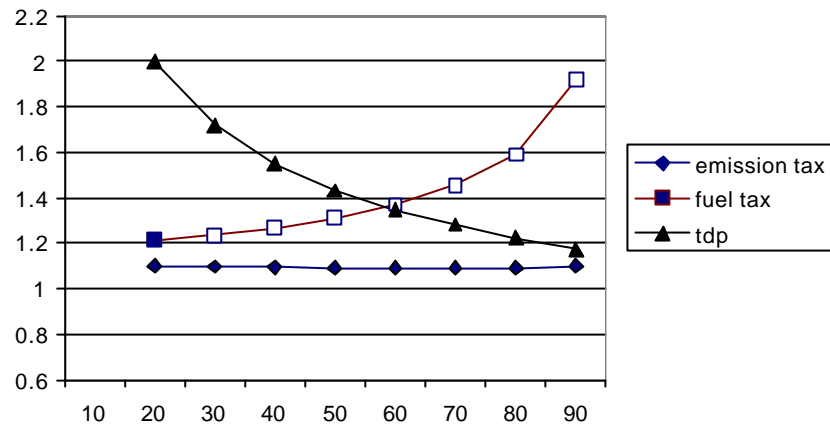

Figure 3: The cost ratio of first-best policy alternatives to the cost of a first-best emission tax without monitoring and enforcement

It is also interesting to compare the gross cost of emission reduction with and without perfect compliance. The gross costs differ because of the total enforcement costs and because of the induced effects on firms' behaviour.

5.2   Second-best setting

We will now take into account the effect of pre-existing distortionary labour taxes ($t_L = 0.4$).
The resulting cost ratios are represented in Figure 4. Again we have results for three policy
instruments: the emission tax, the fuel tax and tradable permits. We compare the case with
(see figure 5) and without perfect compliance (figure 4).

Figure 4: The cost ratio of second-best policy alternatives to the cost of first-best emission tax.
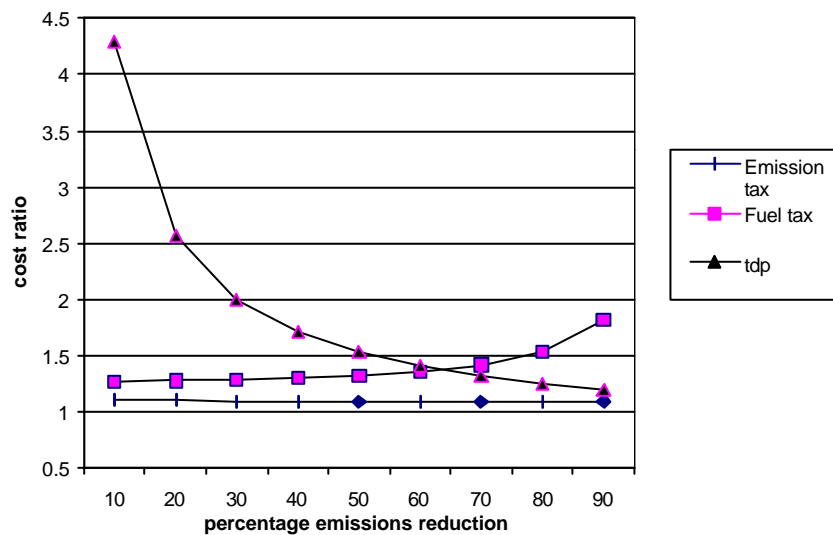
Figure 5: The cost ratio of second-best policy alternatives to the cost of the first-best emission tax in the full compliance case.

We concentrate on the cost curves for imperfect compliance in a second-best world shown in
figure 4. They are all expressed relative to the cost of using emission taxes in a first-best
world. We notice two important effects of bringing in imperfect compliance. First when we

examine the cost of using emission taxes, the cost increase due to the labour market distortion is approximately the same. Secondly, the gross costs of tradable permits are much higher than the costs of emission taxes in a second-best world. The disadvantage of tradable permits has, however, been decreased from 288% to 81% due to the revenue recycling effect that now exists for tradable permits too.

Imperfect compliance does not affect the relative disadvantage of fuel taxes compared to emission taxes.

6.   The role of the fine and the inspection costs

In this section we want to give an indication of the sensitivity of the model to changes in the monitoring and enforcement parameters. The impact of increasing the rent or penalty payment r and the enforcement costs (v and *FIX*) on the results in a first-best setting is given in Table 3.

It is impossible to double the actual fine because the level of the fine depends on the level of the violation. Therefore we consider the impact of a higher penalty payment r on the results. We only look at the emission tax policy and consider a ten percent reduction of emissions.

	Benchmark	Rent payment r (x2)	Enforcement costs v and fix (x2)
Emission tax	1.484359	↑ (1.484376)	↓ (1.483857)
Reported emissions	13105.110	↑ (14015.187)	=
Actual emissions	24026.035	=	=
D	5680.444	↓ (5680.440)	↑ (5680.541)
N	7487.511	↑ (7487.513)	↓ (7487.463)
CD	10441.599	↑ (10441.600)	↓ (10441.551)
CN	416.482	=	↓ (416.480)
Prob. of detection	0.4545	↓ (0.417)	=
Fine[27] D	4215.909	↑ (4215.957)	↓ (4214.556)
Government revenues	27557.977	↑ (27558.304)	↓ (27548.660)
Utility	2923532	↑ (2923536)	↓ (2923434)

[27] We only give the result for the fine of sector D because it is representative for the behaviour of the other sectors.

**Table 3: Sensitivity results for the enforcement parameters for the emission
tax (first-best)**

When the monitoring and enforcement costs for a give inspection level become more
expensive for the government, then we see that the utility decreases. The emission tax
decreases because with a lower tax the firms are more inclined to comply. The government is
therefore able to lower the fine, and the monitoring and enforcement costs, in order to obtain
the same degree of compliance. As indicated in Table 3 an increase in the penalty *r* gives the
firm an incentive to report a larger fraction of their actual emissions to the government.
Consequently the probability of detection decreases[28]. The expected fine will therefore also
decrease in equilibrium. The government must now increase the emission tax in order to
obtain the same emission reduction. The deterrence effect of the fines is decreased and must
be compensated by an increase in the emission tax. The overall effect is an improvement of
social welfare due to the fact that firm behaviour is closer to the full compliance behaviour.
The government can use the less costly instrument of the emission tax to deter firms instead
of using the costly monitoring and enforcement policy.

	Benchmark	Rent payment r (x2)	Enforcement costs v and fix (x2)
Emission tax	1.508769	↑ (1.508804)	↓ (1.507755)
Reported emissions	11290.923	↑ (14015.187)	=
Actual emissions	20700.025	=	=
D	4890.006	↓ (4890.000)	↑ (7890.175)
N	6453.003	↑ (6453.006)	↓ (6452.919)
CD	8998.113	↑ (8998.119)	↓ (8998.034)
CN	358.900	=	↓ (358.897)
Prob. of detection	0.4545	↓ (0.417)	=
Fine D	3688.945	↑ (3689.027)	↓ (3685.593)
Labour tax	0.395867	↓ (0.395865)	↑ (0.395928)
Utility	2849895	↑ (2849900)	↓ (2849772)

**Table 4: Sensitivity results for the enforcement parameters for the emission
tax (second best)**

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[28] Remember that the probability of detection is an increasing function of the size of the violation.

In Table 4 we find the results for a second-best setting. In this setting the labour tax rate is variable since it is assumed that the revenues from the environmental policy are use to reduce the distortionary tax.

When we look at the impact of an increase in the penalty r, we see that social welfare increases. Just as in the first-best setting, the emission tax increases. We can apply the same reasoning to the second-best setting as to the first-best setting. The only difference is the level of the labour tax. Since the emission tax increases it is logical that the labour tax is decreased. There are more environmental revenues that can be used to reduce the labour tax distortion.

The doubling of the monitoring and enforcement costs has the same effect in a second-best setting as in the first-best setting. Social welfare will decrease again. Since the emission tax has decreased, there are less revenues that can be used to reduced the labour tax. Therefore the labour tax is higher than in the benchmark model.

Up to now we have worked with monitoring and enforcement costs functions that are identical for the three instruments considered. One can expect that inspection and enforcement costs will be different when other types of instruments (such technology standards) are considered. This can have important effects for the choice of instruments that do not show up in the Goulder et al. (1999) paper.

7.   Conclusion

In this paper we have added monitoring and enforcement considerations to the literature on environmental policies with distortionary taxes. Our model extends the general equilibrium model of Goulder et al. (1999) to compare the efficiency costs of different policy instruments.

Given the efficiency costs in a full-compliance setting, we can conclude that costs are smaller for all instruments if we work in a setting with incomplete compliance. The reason for this is that we only consider a marginal increase of the *nominal* level of the environmental instrument. However, the economic agents react to the *effective* policy level and not the nominal one. In the model with monitoring and enforcement this effective tax rate is smaller than the nominal one[29]. Therefore the gross efficiency costs of the marginal change in the policy are reduced. However, one should not forget that not only the efficiency costs decrease but also the marginal benefits of the policy. This implies that the environmental policy is more expensive in total if we take imperfect compliance into account.

Imperfect compliance has an important effect on the relative efficiency of tradable emission permits in a second-best setting. In a perfect compliance, second-best framework, tradable permits have a large efficiency disadvantage compared to emission taxes. This disadvantage is strongly reduced as tradable permits now also create public revenue effects and therefore a revenue recycling effect appears. As there is always some noncompliance, there is always some fine revenue and this is used to reduce the existing labour market distortions. This term is positive and reduces therefore the efficiency disadvantage of the tradable permit system found by Goulder et al..

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[29] In the full-compliance model the nominal policy level equals the effective level.

In the numerical model we calculate the relative cost differences for three pricing instruments. We consider emission taxes, fuel taxes and grandfathered tradable permits. Noteworthy is that the relative inefficiency of tradable permits vis-à-vis emission taxes in a second-best setting is decreased when we take the monitoring and enforcement policy into account. This is caused by the presence of the revenue-recycling effect.

The relative inefficiency of the output (fuel) tax remains the same because for both the fuel tax and the emission tax all the terms of the gross efficiency costs are multiplied by the same factor. This factor does not show up when we look at relative cost ratio.

This is a first attempt to integrate monitoring and enforcement considerations into the choice of policy instruments in a second-best setting. Much work still needs to be done.

One obvious extension is to include quantity instruments into the calculations. This will involve using some other assumptions for the expected fine function and this poses new challenges for the numerical model.

Next also broader monitoring and enforcement policy options need to be considered. Fines can be replaced by firm closure, imprisonment, … Since these other penalties are costly for society, while a fine is only a money transfer, the government will have more costs associated with the monitoring and enforcement strategy. Also the assumptions on the probability of detection function can be altered.

Further it would be useful to take the possibility of measurement errors into account. In reality the measurement equipment of the inspection agency is not perfectly accurate. This means that there are false positives and negatives. Some violators are undetected even if they were inspected, while some innocent firms will be sentenced. So there is a distinction between the probability of inspection and the probability of detection.

Another possible extension is to distinguish between the probability of detection and the probability of punishment. In practice we often see that minor violations are left unpunished. This is due to the fact that convicting a firm is not for free. So it is possible that the judge decides to drop the case because it is not worth the time and money to follow up. Therefore the firms will make their decisions based on the probability of punishment and not the probability of detection.

Further extensions can consist of changing the assumptions of the economic model. We could work with heterogeneous firms per sector. Or we could incorporate heterogeneous consumers and take distributional aspects into account. Finally we would introduce imperfect compliance for the labour tax too. This is not unrealistic as the shadow economy counts for 10 to 25% of GDP in Western economies.

Appendix 1: Some calculations

Part 1

We have, using the definition $M \equiv \dfrac{-t_L \dfrac{\partial L}{\partial t_L}}{L + t_L \dfrac{\partial L}{\partial t_L}}$ (13), that:

$$\frac{1+M}{L} = \frac{1 + \dfrac{-t_L \dfrac{\partial L}{\partial t_L}}{L + t_L \dfrac{\partial L}{\partial t_L}}}{L} = \frac{L + t_L \dfrac{\partial L}{\partial t_L} - t_L \dfrac{\partial L}{\partial t_L}}{\left(L + t_L \dfrac{\partial L}{\partial t_L}\right) L} = \frac{1}{L + t_L \dfrac{\partial L}{\partial t_L}}$$

Therefore $\dfrac{dt_L}{dt_E} = -\dfrac{\left(\dfrac{3+4r}{4+4r}\right)\left(E + t_E \dfrac{dE}{dt_E}\right) + t_L \dfrac{\partial L}{\partial p_X}\dfrac{dp_X}{dt_E}}{L + t_L \dfrac{\partial L}{\partial t_L}}$

$$= -\frac{(1+M)}{L}\left\{\left(\frac{3+4r}{4+4r}\right)\left(E + t_E \frac{dE}{dt_E}\right) + t_L \frac{\partial L}{\partial p_X}\frac{dp_X}{dt_E}\right\}$$

Part 2

We are interested in the gross effect of the tax increase t_E on welfare. Differentiating utility $V = v(p_X, 1 - t_L, G + p) - f(E)$ (4) with respect to t_E and ignoring the terms in ϕ, gives:

$$\frac{dv}{dt_E} = \frac{\partial v}{\partial p_X}\frac{dp_X}{dt_E} + \frac{\partial v}{\partial t_L}\frac{dt_L}{dt_E}$$

Using Roy's identity and $\dfrac{dp_X}{dt_E} = e_o - a$ (14), with λ the marginal utility of income gives:

$$\frac{dv}{dt_E} = -\mathbf{l}\, X(e_o - a) - \mathbf{l}\, L\frac{dt_L}{dt_E}$$

$$-\frac{1}{\mathbf{l}}\frac{dv}{dt_E} = X(e_o - a) + L\frac{dt_L}{dt_E}$$

and using $E = (e_0 - a)X$ gives:

$$-\frac{1}{\mathbf{l}}\frac{dv}{dt_E} = E + L\frac{dt_L}{dt_E}$$

Appendix 2: The numerical model

The numerical model was written in GAMS.

This is the description of the model when an emission tax is levied.

1. Sets

$i = \{D, N, L\}$ - inputs
$j = \{D, N, CD, CN\}$ - outputs
$k = \{leis, C, CD, CN\}$

2. Parameters

s_j	elasticity of substitution in production of good j
	/D 0.8, N 0.8, CD 0.9, CN 0.9/
s_c	elasticity of substitution between consumption goods
	/0.85/
b_i	pollution content of good i used
	/D 0.062768, N 0, L 0/
a_e	effectiveness of abatement technology
	/0.155/
g	curvature parameter for abatement
	/0.5/
t_r	rent taxation rate
	/0.4/
t_L	labour taxation rate
	/0.4/
T	total time endowment
	/2129897.1/
reduc	emission reduction
Etotmax	maximum amount of emissions possible (in millions of kg)
	/23000.028/
a_{ij}	distribution parameter for input i in production of good j (via calibration)
a_k	distribution parameter for the utility function (via calibration)
Fix	fixed costs associated with the monitoring and enforcement policy
v	variable costs associated with the monitoring and enforcement policy

3. Variables

b_{ij}	use of input i per unit of output of good j

b_{CD} , b_{CN} relative share of consumption of CD and CN respectively to total consumption

CDdem   aggregate demand for energy-intensive goods

CNdem   aggregate demand for non-energy intensive final goods

AD_i aggregate demand for good i

X_j aggregate supply of good j

L aggregate labour supply

C aggregate demand for composite consumption good

leis   leisure or non-market time

$\boldsymbol{p}_j$ profit per industry j

$\boldsymbol{p}_{tot}$ total profits or total pollution quota rents

p_j price of output j

pr_i price of input i

p_C price of composite good

REV   government revenue

$\boldsymbol{t}_e$ emission tax

$\boldsymbol{t}_j$ tax on output j

G lump-sum transfer

E_j actual pollution emitted from production of good j

Er_j reported emissions from production of good j

Etot   total actual emissions

Ertot   total reported emissions

A_j abatement expenditure in industry j

U total consumer utility

X_{ij} use of good i in production of good j

$p_{\det j}$ probability of detection per sector j

F_j fine per sector j

r interest rate to be paid on overdue taxes

4. Equations

4.1 Production – firm behaviour

Output

$$X_j = \boldsymbol{d}\left(\sum_i \boldsymbol{a}_{ij} X_{ij}^{\frac{s_j-1}{s_j}} \right)^{\frac{s_j}{s_j-1}}$$

Profit

$$p_j = (p_j - t_j)X_j - \sum_i p_i X_{ij} - t_e Er_j - A_j - p_{\det j}.F_j$$

Total profits

$$\boldsymbol{p}_{tot} = \sum_j \boldsymbol{p}_j$$

Expected fine

$$p_{\det j}.F_j = \frac{\left(E_j - Er_j\right)}{E_j}.\left(E_j - Er_j\right)\boldsymbol{t}_e\left(1 + r\right) = \frac{3 + 4r}{4 + 4r}\boldsymbol{t}_e E_j$$

Emissions

$$E_j = \boldsymbol{b}_D X_{Dj}\left[1 - \boldsymbol{a}_e\left(\frac{A_j}{\boldsymbol{b}_D X_{Dj}}\right)^{\boldsymbol{g}}\right]$$

Total emissions

$$Etot = \sum_j E_j$$

Reported emissions

$$Er_j = \frac{1 + 2r}{2 + 2r}E_j$$

First-order condition (A_j)

$$A_j = \boldsymbol{b}_D X_{Dj}\left(\frac{3 + 4r}{4 + 4r}\boldsymbol{t}_e \boldsymbol{a}_e \boldsymbol{g}\right)^{\frac{1}{1-\boldsymbol{g}}}$$

First-order condition (X_{ij})

$$b_{ij} = \frac{X_{ij}}{X_j} = \frac{\boldsymbol{a}_{ij}^{\,\boldsymbol{s}_j}}{\boldsymbol{d}}\left\{\frac{pr_i + \boldsymbol{b}_i \boldsymbol{t}_e\left(\frac{3 + 4r}{4 + 4r}\right)\left[1 - \boldsymbol{a}_e\left(\frac{A_j}{\boldsymbol{b}_D X_{Dj}}\right)^{\boldsymbol{g}}(1 - \boldsymbol{g})\right]}{p_j - \boldsymbol{t}_j}\right\}^{-\boldsymbol{s}_j}$$

Price

$$p_j = \boldsymbol{t}_j + \sum_i b_{ij} pr_i + \frac{\left(\frac{3 + 4r}{4 + 4r}\right)\boldsymbol{t}_e E_j + A_j}{X_j}$$

4.2 Household behaviour

Utility

$$U = U\left(l, CD, CN, E\right) = \left(\boldsymbol{a}_{leis}\, leis^{\frac{\boldsymbol{s}_u - 1}{\boldsymbol{s}_u}} + \boldsymbol{a}_C\, C^{\frac{\boldsymbol{s}_u - 1}{\boldsymbol{s}_u}}\right)^{\frac{\boldsymbol{s}_u}{\boldsymbol{s}_u - 1}}$$

Composite consumption (1)

$$C = h \left(a_{CD} \, CD^{\frac{s_C - 1}{s_C}} + a_{CN} \, CN^{\frac{s_C - 1}{s_C}} \right)^{\frac{s_C}{s_C - 1}}$$

Budget constraint

$$p_{CD} CD + p_{CN} CN = pr_L (1 - t_L) L + p \;\; (1 - t_R) + p_C \;\; G$$

Distribution

$$b_{CD} = \frac{CD}{C} = \frac{1}{h} \left[a_{CD} + a_{CN} \left(\frac{a_{CD} \, p_{CN}}{a_{CN} \, p_{CD}} \right)^{1 - s_c} \right]^{\frac{s_c}{s_c - 1}}$$

$$b_{CN} = \frac{CN}{C} = \frac{1}{h} \left[a_{CN} + a_{CD} \left(\frac{a_{CN} \, p_{CD}}{a_{CD} \, p_{CN}} \right)^{1 - s_c} \right]^{-\frac{s_c}{s_c - 1}}$$

Composite price

$$p_C = p_{CD} b_{CD} + p_{CN} b_{CN}$$

Leisure

$$leis = \frac{pr_L (1 - t_L) T + p_C G + (1 - t_R) p}{pr_L (1 - t_L) + p_C \left[\dfrac{a_{leis} \, p_C}{a_c \, pr_L (1 - t_L)} \right]^{-s_u}}$$

Labour

$$L = T - leis$$

Numeraire

$$pr_L = 1$$

Composite consumption (2)

$$C = p_C^{-1} \left[pr_L (1 - t_L) + p_C G + (1 - t_R) p \right]$$

4.3 Government behaviour

Revenue

$$REV = t_L L + t_e Ertot + \sum_j t_j X_j + t_r p_{tot} + \sum_j p_{\det j} . F_j$$

Tax

$$t_r = t_L$$

4.4 Aggregate demand and supply

Inputs

$$AD_i = \sum_j X_{ij}$$

Outputs

$$X_{CD} = CD$$
$$X_{CN} = CN$$
$$X_i = AD_i \qquad for \; i = \{D, N\}$$

4.5 Equilibrium conditions

Labour market

$$L = AD_L + \sum_j A_j$$

Emissions

$$Etot = (1 - reduc)\,Etotmax$$

Government

$$REV = p_C G$$