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STOCHCLIM

Improving the representation and prediction of climate processes through stochastic parameterization schemes

FINAL REPORT

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ABSTRACT

Context

A wide variety of climate-type phenomena have been discovered from monthly time scales like the Madden-Julian Oscillation (MJO) up to decadal time scales like the North Atlantic Oscillation (NAO) circulation, which potentially have important impacts for medium term social and economic planning. Currently, large (modeling) uncertainties hamper the possibility to simulate and forecast accurately these medium to long term processes in a satisfactory manner.

Objectives

The central goal of the project was to improve the understanding and description of key physical processes in climate models of increasing complexity, with emphasis on the improvement of the variability of dynamical phenomena acting on daily, monthly, seasonal, decadal and longer time scales. This aim has been pursued by developing and assessing new parameterization (closure) schemes incorporating stochastic components. To achieve this goal, four specific questions were addressed,

- (i) What type of parameterization schemes including stochastic components (and where) should be introduced in climate models in order to improve their variability, in particular at seasonal and decadal time scales? This question has been addressed by developing, calibrating and assessing stochastic schemes in low-order and intermediate order climate models.
- (ii) What is the dynamical response of stochastic climate models to slow transient forcings, and what are the precursors of abrupt climate transitions? A theoretical analysis of the impact of stochastic processes in simple nonlinear dynamical climate systems in the presence of slow transient forcings and model errors has been performed.
- (iii) What is the usefulness of introducing stochastic components in current convection parameterization schemes of detailed climate models in order to improve the quality of their statistical properties, in particular for the water cycle? The investigation mainly focused on new parameterization (convection) schemes developed at RMI, and their stochastic extensions, incorporated in the regional operational model ALARO.
- (iv) To what extent stochastic parameterization improves the information on the forecast uncertainties of climate models? This question has been addressed in the context of the models already mentioned above, with emphasis on our ability to track processes acting from seasonal up to decadal time scales.

The fundamental aspects of stochastic physics for the correction of model errors and the quality of forecast were implemented through the cross-fertilization of theory and practice.

Conclusions

The project led to the development and analysis of stochastic schemes in a variety of models, from low-order and intermediate-order climate models up to a detailed Numerical Weather Prediction system. In these different contexts, these schemes were found to provide very encouraging results. On the theoretical side, the most efficient scheme implemented in the low- to intermediate-order systems turned out to be the homogenization method, which should now be extended to more comprehensive climate models. Although there is no a priori theoretical limitation in the implementation of this scheme, it necessitates a careful investigation of the variability of sub-grid scale processes that could be very cumbersome to get an operational version of this scheme in a detailed climate models. This is a long-term goal on its own, that will potentially revolutionize the current way to perform climate modelling. On a more empirical side, very encouraging results were obtained on the development of a stochastic scheme of the momentum fluxes in a convection permitting model using an approach inspired from the dynamics of model errors. This approach should now be extended to take into account spatio-temporal correlations between the perturbations and the errors in the initial conditions, and eventually tested in an operational environment.

Several statistical and dynamical tools have been identified to monitor the occurrence of potentially catastrophic climate changes (bifurcations) in the context of generic simplified low-order atmospheric and climate models. In particular, climate responses to time varying perturbations such as the atmospheric CO₂ concentration were analysed. In contrast to typical climate model experiments in which the CO₂ concentration is instantaneously doubled or quadrupled, the CO₂ concentration was here taken to increase smoothly in time, as it is expected to be the case in reality. Substantial differences with standard scenario were brought out. These tools should now be implemented and tested in the context of comprehensive climate models and confronted with data.

Besides its scientific success, the project also contributed to consolidate the position of Belgian scientists at the international level.

Keywords

Climate models, Low-Frequency Variability, Stochastic modelling, Calibration, Climate Change

1. INTRODUCTION

One of the main features of climate is the presence of variability on a wide range of time scales, from weeks to decades and even millennia. A variety of phenomena were discovered like the Madden-Julian Oscillation (months), the North Atlantic Oscillation (season), the El-Niño-Southern Oscillation (ENSO, years), the Quasi-Biennial Oscillation (years), the Atlantic and Pacific multi-decadal oscillations (decades), see Hoskins (2012). A great deal of effort has been devoted to simulate these phenomena with climate models of various resolutions and complexity. Modelling these multiple time scale processes is however a formidable task far from being completed. For instance, the stability of the overturning circulation is largely model dependent, and constitutes a source of uncertainty at time scales of 10 to 100 years.

Despite the chaotic nature of the atmosphere and climate on short time scales (Vannitsem and Nicolis, 1997, 1998), more predictable phenomena appear to emerge on longer time scales. From a climatic perspective, this variability is a natural extension of the short atmospheric variability and it could be predicted with similar methods. This has led to the idea of developing systems of climate prediction on seasonal, annual and decadal time scales based on an approach similar to the one used for weather forecasts. Hence, the notion of seamless weather and climate prediction (and simulations) valid at all timescales is born (e.g. Hoskins, 2012). Clearly this approach suffers from two conflicting objectives: on the one hand the necessity to improve the quality of the models through the increase of resolution and dynamical processes description up to a cloud-resolving scale or less (of the order of a kilometer); on the other hand, the necessity of providing predictions or simulations on longer and longer timescales. As underlined in Slingo and Palmer (2012), a trade-off should be found between the details of description of processes (i.e. the presence of model errors) and the affordability of climate forecasts or projections.

In addition, model uncertainties become increasingly crucial as the forecast time increases. These uncertainties can be split into two main categories, (i) the ones arising from the imperfect knowledge of the real system such as for instance the carbon cycle or the interaction between the ocean and the atmosphere (Vannitsem and Toth, 2002; Nicolis 2003), (ii) the ones associated with sub-grid scale processes that are not effectively resolved by the models (Nicolis, 2004; Tribbia and Baumhefner, 2004). Superimposed to these, a portion of uncertainty is related to the natural variability of the dynamics - associated with its chaotic nature - that depends on the lead time of the projection. This repartition between model errors and natural variability has been nicely illustrated in the 2009 UK Climate Projections (<http://ukclimateprojections.defra.gov.uk>); see also Slingo and Palmer (2012). A theory of the dynamics of the combination of both types of errors has been developed in Nicolis et al (2009) and it has been shown that the error growth is dominated by the initial condition errors at short time scales, while model errors are taking over at longer time scales.

Traditionally, the influence of processes that are not explicitly resolved by a model is expressed through parameterization schemes in which this influence is expressed in terms of the variables effectively present in the model. This approach has been very successful in

particular in the domain of operational Numerical Weather Prediction (NWP). In the following, we will refer to these approaches as “deterministic” parameterization since they do not introduce any random effects.

In recent years however, many approaches have been proposed to address the problem of model uncertainties in Numerical Weather Prediction through the use of new sub-grid scale parameterization schemes based on stochastic approaches (see the workshop proceedings of the ECMWF, “Representing model uncertainty and error in numerical weather and climate prediction models”, 2011).

Stochastic processes were originally introduced to represent rapidly evolving processes not resolved by the model at hand as random forcings with prescribed statistical properties (Hasselmann, 1976, Nicolis, 1982). At that time these stochastic forcings were viewed as environmental perturbations superimposed on the process of interest. The approach has then evolved and been used extensively to model processes not explicitly present in the model of the system under investigation. A recent example is the use of stochastic wind forcings to drive the upper surface layer of the ocean (e.g. Ivanov and Chu, 2007).

The current paradigm views stochastic schemes either as one approach to the problem closure of sub-grid scale processes (for example, turbulence) or as a method to quantify the effects on unknown processes. In both cases, they lead to a practical approach to provide a fully probabilistic description of the evolution of the weather and climate dynamics, with a robust estimate of uncertainties. The introduction of this new approach opened the debate about what should be the very nature of a parameterization scheme (determinist or stochastic) and for what kind of processes. For instance, it is believed that radiative schemes should not be stochastic per se, but the cloud scheme interacting with it should be stochastic (optical properties, cloud fraction...), Pincus (2011).

2. STATE OF THE ART AND OBJECTIVES

The central goal of the project was to improve the understanding and description of key physical processes - in particular the water cycle in Numerical Weather Prediction models. Emphasis was set on the improvement of the variability of dynamical phenomena acting from monthly, seasonal, decadal up to centennial time scales with special attention paid to processes associated with the presence of Low-Frequency Variability (LFV). To reach this objective, the investigators were developing new parameterization (closure) schemes incorporating stochastic components, and assessing their impact on the mean, variance and higher order moments of the climate system variables, together with their LFV. In particular, four specific questions were addressed,

- (i) What type of parameterisation schemes including stochastic components should be introduced in climate models in order to improve their variability, in particular at seasonal and decadal time scales? This question was addressed by developing, calibrating and assessing stochastic schemes in low-order and intermediate order climate models, as described in more detail in Sections 4.2, 4.3 and 4.4.
- (ii) What is the dynamical response of stochastic climate models to slow transient forcings such as the increase of greenhouse gases, and what are the precursors of abrupt climate transitions? A theoretical analysis of the impact of stochastic processes in simple nonlinear dynamical climate systems in the presence of slow transient forcings and model errors has been performed, as discussed in Sections 4.5 and 4.6.
- (iii) What is the practical impact of introducing stochastic components in current convection parameterization schemes of detailed climate models on the quality of their statistical properties, in particular for the water cycle? The investigation has focused on new parameterization (convection) schemes developed at RMI, and their stochastic extensions, incorporated in the regional operational model (ALARO). The results are discussed in Section 4.7.
- (iv) To what extent stochastic parameterisation improve the quantification of forecast uncertainties of climate models? This question will be addressed in the context of the models already mentioned above, with emphasis on our ability to track processes acting from days to millennia. Section 4.8 discusses the specific analysis made in the context of convective processes in NWP.

The use of stochastic physics for the correction of model errors and the quality of forecast were implemented through the cross-fertilization of theory and practice. These process-oriented investigations, drawing on an array of complementary approaches, allow for a reduction of robust biases known to be present in model climatologies, and for a more precise quantification of the forecast uncertainties. Potential abrupt transitions (and their precursors) under the presence of stochastic forcings were also explored in simplified representative models of the atmospheric and climate dynamics.

Interestingly, as it should be in science, other activities not directly related to the main topics originally proposed in the project were explored. First the development of a flexible coupled ocean-atmosphere model allowing for a detailed understanding of the low-frequency variability of the atmosphere and for the test of different techniques such as stochastic modelling. Second, the strategy consisting in calibrating stochastic dynamical systems to match observations generated significant insights into the dynamics of past abrupt events.

These questions were addressed using the complementary expertise of the different teams involved in the project, namely the knowledge on stochastic processes and dynamical system theory (RMI and UCL), the dynamics and physics of intermediate order and detailed atmospheric models (RMI, UCL, U Gent).

3. METHODOLOGY

The basic tools used are coming from theories of nonlinear dynamics and stochastic processes, relevant in the context of weather and climate forecasts (e.g. Nicolis, 2005). These tools were developed and used in the context of a hierarchy of climate models, from low-order coupled ocean-atmosphere models (e.g. De Cruz et al, 2016; Vannitsem et al 2015) to more detailed climate models (e.g. Goosse et al, 2010; ALADIN Team, 1997), available at RMI, Ugent and UCL. Their impact was inferred by comparing with the statistical and dynamical properties of the reference climate. Several appropriately-chosen approaches were implemented in the context of a hierarchy of models and their variability compared with the natural one. More specifically,

1. Low-order models are prototypical representation of some dynamical processes present in the climate system. These models are natural test-beds for the analysis of quality of new tools and techniques that are then applied in the context of more complex systems. Different stochastic strategies and their optimization were tested in the context of low order systems displaying multiple time scales, the low-order coupled ocean-atmosphere (Demaeyer and Vannitsem, 2018), in paleoclimate models (Mitsui and Crucifix, 2017), and in prototypical systems of atmospheric and climate processes (e.g. Nicolis, 2018).
2. The LOVECLIM model developed by the UCL. LOVECLIM is a coupled ocean-atmosphere model of intermediate complexity (Goosse et al, 2010). One version of this model is now implemented at RMI. This model displays a variability that is too small as compared with the natural one. In this context, stochastic perturbations of convective processes in the ocean were tested.
3. The HARMONIE system is a model code that is shared with the IFS system of ECMWF allows to run different model configurations with various physics packages. For instance the ALARO physics was designed to be used at resolutions of 3 to 5 km, the AROME configuration is aimed at 2 km resolutions and there is also a global version called ARPEGE developed by Meteo France. The model configurations are operationally used for Numerical Weather Prediction. Recently tests have been carried out with running ALARO on the globe. Within the ALARO model stochastic physics have already been implemented based on the idea of Cellular automata (L. Bengtsson 2011). This technique is however empirical and processes-based approaches are called for. Tests with stochastic convection schemes were performed based on the concept of model error developed in Nicolis (2003).

During the last century, the increase of greenhouse gases in the atmosphere has introduced an additional slow varying radiative forcing. This slow variation modifies the statistical and dynamical properties of the Earth system as reflected in many detailed climate model runs, see for instance in the IPCC report (2007). However, the response to such a slow forcing is highly dependent on the underlying structure of the system's attractor, in particular in the

presence of noise (e.g. Nicolis, 1988). This problem was further explored here by analysing some specific bifurcations (pitchfork, limited point), known to be present for instance in oceanic flows (Dijkstra, 2005). The impact of slowly varying forcings was also explored in the context of low order chaotic systems (Vannitsem, 2015).

In addition, the question arises concerning our ability to predict the occurrence of potential climate transitions. This question has notably been explored in the ECBILT-CLIO model, the ancestor of LOVECLIM and it was found, using numerical experiments, that the predictability of climate transition in the North Atlantic was quite limited due to the atmospheric variability affecting the coupled model (Schaeffer et al, 2002). This question was addressed here in the context of simple generic systems describing bifurcations (pitchfork, limit point) by investigating the potential precursors of these climate changes.

Finally the implications of our approach to the predictability problem of medium term forecasts (in the presence or not of transient forcing and climate change) were assessed, with special focus on the seasonal, inter-annual and decadal time scales.

4. SCIENTIFIC RESULTS AND RECOMMENDATIONS

4.1 A new general framework to investigate coupled ocean-atmosphere dynamics

The atmosphere at mid-latitudes displays variability on a wide range of space scales and timescales, and in particular a low-frequency variability at interannual and decadal timescales as suggested by the analyses of different time series developed in the past years (Trenberth, 1990). In contrast to the phenomenon of El Niño–Southern Oscillation (ENSO), of which the driving mechanisms are intensively studied and quite well understood, the origin of mid-latitude low-frequency variability (LFV) remains highly debated, mainly due to the poor ability of state-of-the-art coupled ocean–atmosphere models to simulate it correctly (e.g. Smith et al., 2014). The most plausible candidates of this LFV are either the coupling with the local ocean (Kravtsov et al., 2007), or teleconnections with the tropical Pacific ocean–atmosphere variability (Müller et al., 2008), or both.

Recently the impact of the coupling between the ocean and the atmosphere at mid-latitudes on the atmospheric predictability (Peña and Kalnay, 2004) and the development of the LFV (van Veen, 2003) has been explored in a series of low-order coupled ocean–atmosphere systems. However, the limited flexibility of the possible geometries of these previous models led the present authors to develop a series of new model versions. The first of these, OA-QG-WS v1 (Vannitsem, 2014), for Ocean-Atmosphere–Quasi-Geostrophic–Wind Stress, features only mechanical couplings between the ocean and the atmosphere. It contains 12 atmospheric variables following Charney and Straus (1980) and four oceanic modes following Pierini (2011). In a successor of this model, OA-QG-WS v2, the set of atmospheric variables is extended from 12 to 20 as in Reinhold and Pierrehumbert (1982). This increase in resolution in the atmosphere was shown to be key to the development of a realistic double gyre in the ocean (Vannitsem and De Cruz, 2014). A third version of this model, hereafter referred to as VDDG in reference to the authors of the model, includes passively advected temperature in the ocean and an energy balance scheme, combined with an extended set of modes for the ocean (Vannitsem et al, 2015).

In the VDDG model, an LFV associated with the coupling between the ocean and the atmosphere was successfully identified, allowing for extended-range coupled ocean–atmosphere predictions (Figure 1). Moreover, the development of this coupled ocean–atmosphere mode is robust when stochastic forcings are added (Demaeyer and Vannitsem, 2016), or when a seasonal radiative forcing is incorporated into the low-order model (Vannitsem, 2015). Remarkably the presence of the seasonal radiative input favours the development of the coupled mode due to the amplification of the impact of the wind stress forcing in summer, associated with a drastic reduction of the mixed layer thickness at that period of the year.

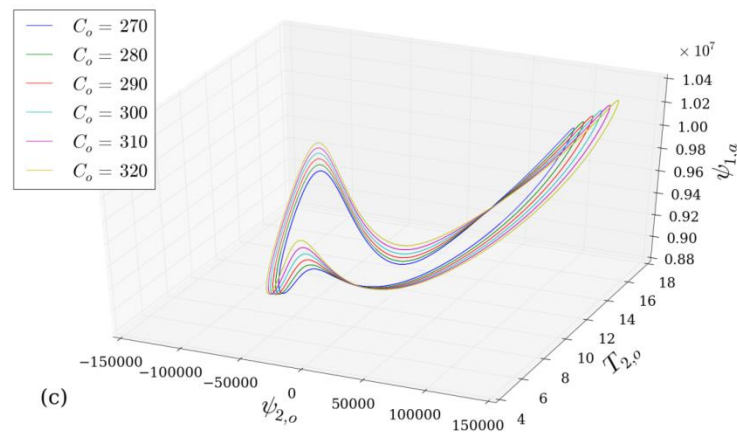


Figure 1. Decadal transport structure found in the new coupled VDDG ocean-atmosphere model for various values of the parameter C_o controlling the amount of energy absorbed by the ocean component.

While these are encouraging results, which suggest the generic character of the coupled ocean–atmosphere mode, they need to be confirmed through the analysis of more sophisticated models, and in particular in higher-resolution coupled systems. In that perspective, a new model that generalizes the VDDG model by allowing for an arbitrary number of modes, or basis functions in which the dynamical fields are expanded. The modes can be selected independently for the ocean and the atmosphere, and for the zonal and meridional directions. The modular approach allows one to straightforwardly modify the model physics, such as changing the drag coefficient, introducing new dissipative schemes or adding a seasonal insolation. This model was coined MAOOAM: the Modular Arbitrary-Order Ocean-Atmosphere Model, available on the platform Github (<https://github.com/Climdyn/MAOOAM>). The dynamical properties of this model have been studied in De Cruz et al (2016, 2018). This framework has been furthermore used to test new stochastic schemes as presented in the next section.

4.2 Stochastic physics viewed as new closure schemes

Climate models are not perfect, as they cannot encompass the whole world in their description. Model inaccuracies, also called model errors, are therefore always present (Trevisan and Palatella, 2011). One specific type of model error is associated with spatial (or spectral) resolution of the model equations. A stochastic parameterization is a method that allows for representing the effect of unresolved processes into models. It is a modification, or a closure, of the time-evolution equations for the resolved variables that take into account this effect. A typical way to include the impact of these scales is to run high-resolution models and to perform a statistical analysis to obtain the information needed to compute a closure of the equations such that the truncated model is statistically close to the high-resolution model, as discussed in Section 4.7. These methods are crucial for climate modeling, since the models need to remain as low-resolution as possible, in order to be able to generate runs for very long

times. In this case, the stochastic parameterization should allow for improving the variability and other statistical properties of the climate models at a lower computational cost. Recently, a revival of interest in stochastic parameterization methods for climate systems has occurred, due to the availability of new mathematical methods to perform the reduction of ordinary differential equations (ODEs) systems. Either based on the conditional averaging (Kifer, 2001; Arnold, 2001; Arnold et al., 2003), on the singular perturbation theory of Markov processes (Majda et al., 2001) (MTV), on the conditional Markov chain (Crommelin and Vanden-Eijnden, 2008) or on the Ruelle response theory (Wouters and Lucarini, 2012) and non-Markovian reduced stochastic equations (Chekroun et al., 2014, 2015).

These methods have all in common a rigorous mathematical framework. They provide promising alternatives to other methods such as the ones based on the reinjection of energy from the unresolved scale through backscatter schemes (Frederiksen et al, 2017) or on empirical stochastic modeling methods based on autoregressive processes (Arnold et al., 2013). The usual way to test the effectiveness of a parameterization method is to consider a well-known climate low-resolution model over which other methods have already been tested. For instance, several methods cited above have been tested on the Lorenz'96 model, see e.g. Crommelin and Vanden-Eijnden (2008); Arnold et al. (2013); Abramov (2015) and Vissio and Lucarini (2016). These approaches have also been tested in more realistic models of intermediate complexity that possess a wide range of scales and possibly a lack of timescale separation¹, like for instance the evaluation of the MTV parameterization on barotropic and baroclinic models (Franzke et al., 2005; Franzke and Majda, 2006). Due to the blooming of parameterization methods developed with different statistical or dynamical hypothesis, new comparisons are called for.

A first exploration has been made in the context of a low-order coupled Ocean-Atmosphere model. The central approach was to reduce a coupled ocean-atmosphere system into a single ocean model forced stochastically. Once this new simplified model was built, its ability to reproduce the correct variability and to forecast the ocean generated by the full deterministic system, have been tested. The stochastic modelling followed closely the approach developed in (Arnold et al, 2003), known as *stochastic averaging*. It provides a systematic approach for reducing a coupled slow-fast system into a stochastic system describing the dynamics of the slow component only. This approach is obviously very much suitable in the problem of modelling forcings of an ocean, as revealed by the results reported in Vannitsem (2014). This is illustrated in Figure 2 in which the probability distribution of one particular variable generated by the full (low-order) ocean-atmosphere system and the one produced by the stochastic ocean model.

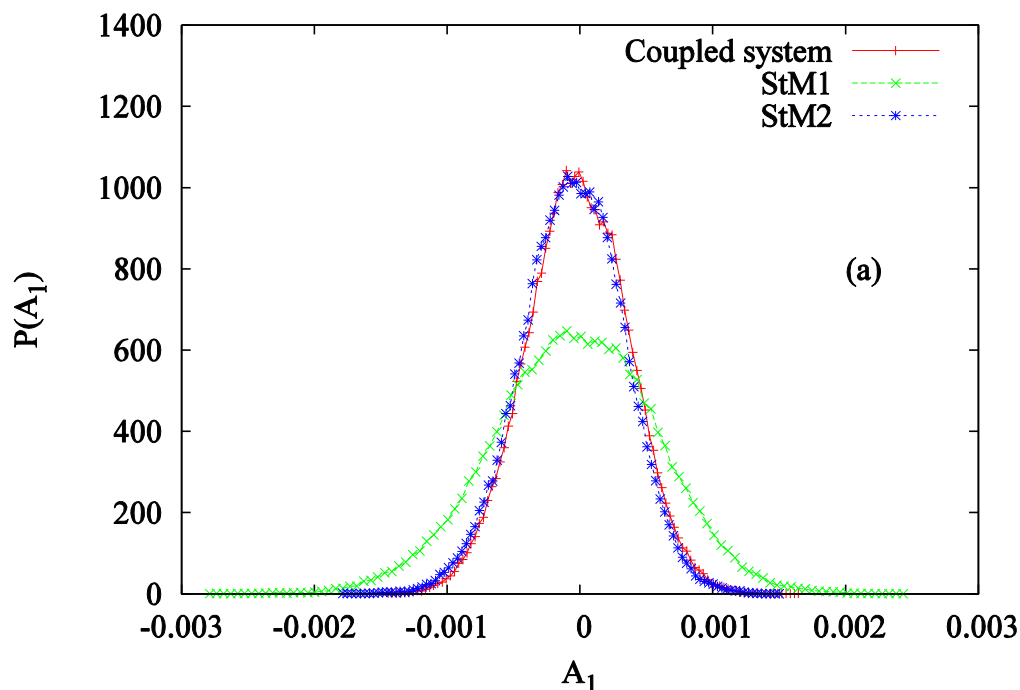


Figure 2. Probability density of the coupled ocean-atmosphere model, and the stochastic model using two different estimates of the parameters, for the first ocean variable. Non-dimensional units are used.

This method, although very rigorous, is difficult to implement in systems with many degrees of freedom due to the necessity of making an average over a measure conditional on the value of the slow variables. Alternative methods should therefore be used allowing for alleviating this problem. Two other approaches have been proposed recently in that context. The first one is based on the singular perturbation of Markov operator, also known as homogenization (Majda et al., 2001). The second one is a recently proposed parameterization based on the Ruelle's response theory (Wouters and Lucarini, 2012).

The two parameterizations were extensively tested for a low-order version of MAOOAM known to exhibit low-frequency variability (see Figure 3), and some preliminary results were also obtained for an intermediate-order version (see Figure 4). Both parameterizations showed remarkable performances in correcting the impact of model errors, and notably being able to change the modality of the probability distributions.

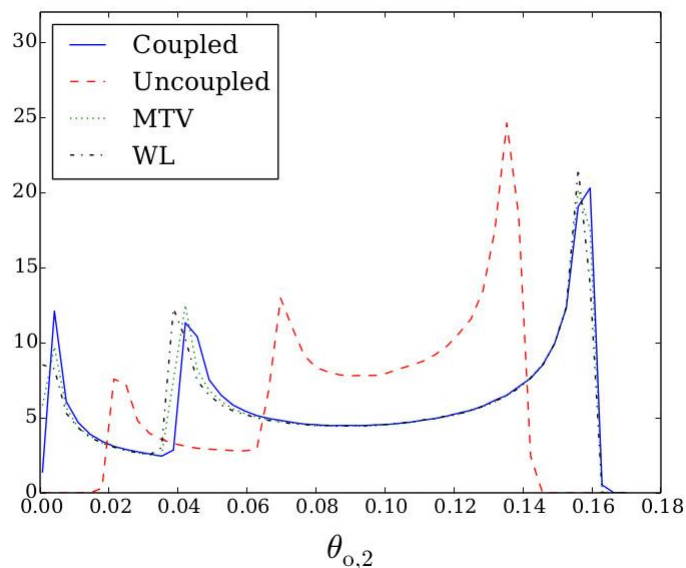


Figure 3: In a low-order version of MAOOAM, correction of the probability density function (PDF) of the dominant oceanic temperature mode. “Uncoupled” is the PDF of the uncorrected system. “Coupled” is the PDF of the “truth”. “MTV” and “WL” are the two correction methods (parameterization). Notice the good agreement between these two latter and the “truth”, showing the effectiveness of the proposed parameterization methods.

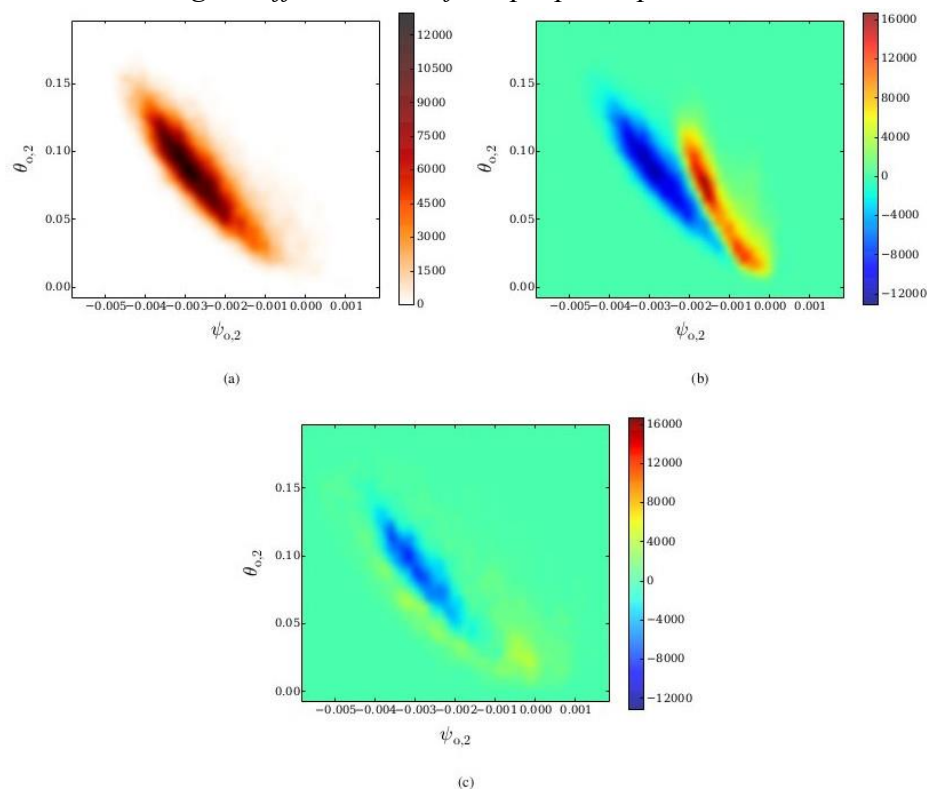


Figure 4: In an intermediate, higher-order version of MAOOAM: (a) two-dimensional “truth” PDF of the two dominant oceanic modes. (b) anomaly of the uncorrected system PDF with respect to the “truth”. (c) anomaly of the “MTV” corrected system with respect to the “truth”. Notice the diminution of the anomaly due to the correction of the parameterization.

In most of the cases studied, the two methods performed well, correcting the marginal probability distributions and the autocorrelation functions of the model variables, even in the cases where a LFV is developing in the model. However, we have also found that the WL method shows instabilities, due to the cubic interactions therein. It indicates that the applicability of this method may crucially depend on the long-term correlations in the underlying system. The MTV method does not exhibit this kind of problem. Additionally, we have found that these methods are able to change correctly the modality of the distributions in some cases. However, in some other cases, they can also trigger a LFV that is absent from the full system. This leads us to underline the profound impact that a stochastic parameterization, and noise in general, can have on models. For instance, Kwasniok (2014) has shown that the noise can influence the persistence of dynamical regimes and can thus have a non-trivial impact on the PDFs, whose origin is the modification of the dynamical structures by the noise. In the present study as well, the perturbation of the dynamical structures by the noise is a very plausible explanation for the observed change of modality and for the good performances of the parameterizations in general. However, if these perturbations can lead to a correct representation of the full dynamics, they can also generate regimes that are not originally present.

4.3 Test of Stochastic schemes in the context of an intermediate order climate model:

LOVECLIM

We introduce and evaluate a stochastic parameterization of deep-ocean convection in the low-resolution climate model LOVECLIM (Goosse et al. 2010). The horizontal discretization of the ocean in LOVECLIM is based on spherical coordinates, using a resolution of 3° by 3° , and the vertical column includes up to 20 levels in the deep ocean. The purpose of the stochastic scheme is to induce (stochastically) convection even when the ocean vertical column is slightly stable, following the argument that convection may occur sporadically even when, on average, the vertical gradient of density is positive downwards. The general objective is to assess the effect of this parameterization on the variability of the model, from interannual to centennial scales.

Technically, convection is prescribed to occur when $N^2 < \varepsilon_i$, where ε_i is a random number distributed as a normal distribution, with mean μ and standard deviation σ . The index i represents the time step, and ε_i follows Gaussian white noise dynamics. The deterministic (standard) scheme is recovered when $\sigma = 0$. As a reference, note that N^2 at great depth in the ocean is of the order of $10^{-8}s^{-2}$, and of the order $10^{-5}s^{-2}$ in the mixed layer. We focus on a specific grid point, representing the temperature North of Norway, which is an active site of deep water formation in this model.

We considered two reference experiments, one with $\sigma = 0$ (deterministic) and one with $\sigma = 2 \cdot 10^{-6}s^{-2}$. The quantity shown on Figure 5 is the annual mean temperature at that point. The two experiments are broadly indistinguishable. The experiment including the stochastic

parameterizations might have slightly more cold events, but this observation does not pass any statistical test (Kolmogorov-Smirnov, Wilcoxon, t-test). The power spectrum of both experiments is also similar (not shown).

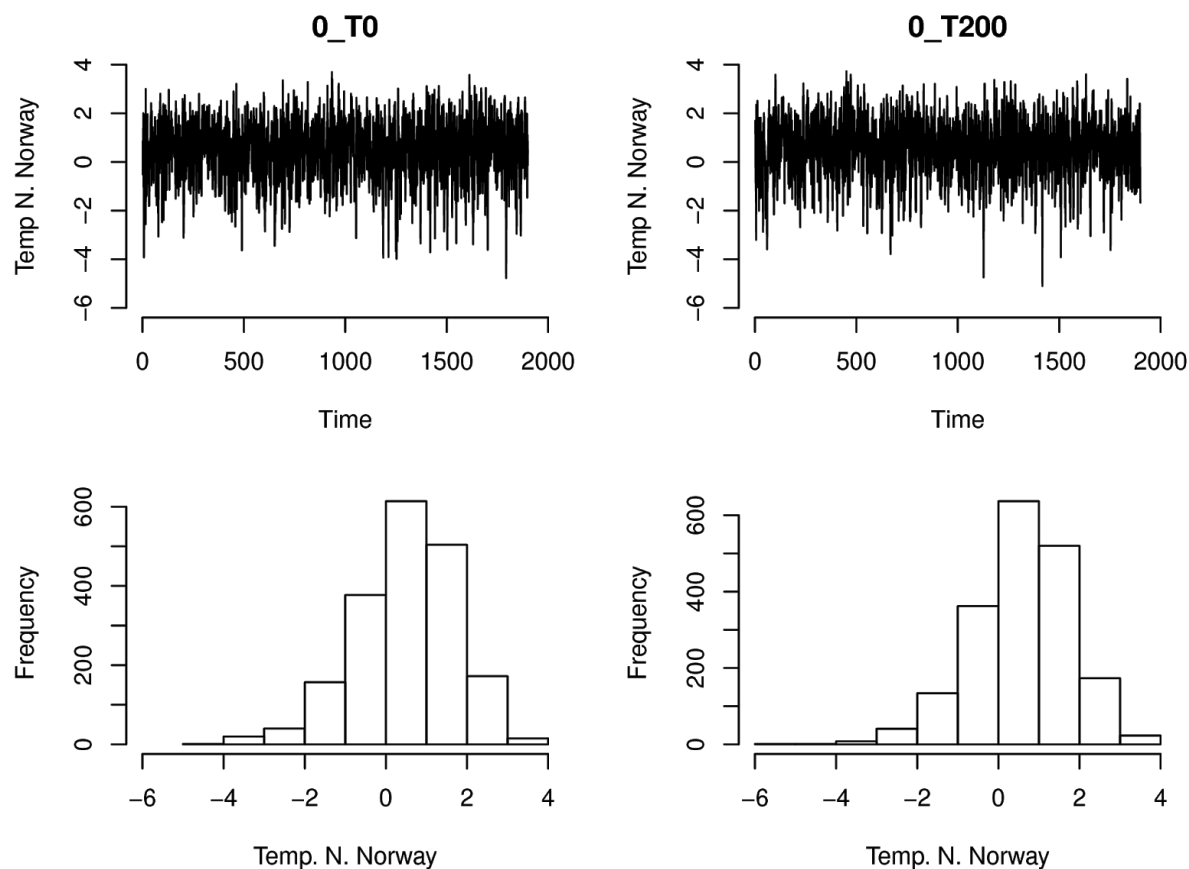


Figure 5: Reference orbital configuration: Temperature North of Norway, without (left) and with (right) stochastic parameterisation

More interesting results have however been found when LOVECLIM is in a self-oscillating regime. LOVECLIM may indeed present intermittent dynamics (reminiscent of a limit cycle) in certain configurations, reached, for example, when the orbital forcing is set to certain values. Such oscillations have been observed in several ocean-atmosphere experiments (Hall and Stouffer 2001). It is generally believed that a regime of intermittent shut-off of the circulation expresses the proximity to a bifurcation yielding either to a permanent shut-off of the circulation, or to the development of a limit cycle known in the literature as a *regime of deep-decoupling oscillations*. However, the oscillation found in 3-D models does not necessarily imply a succession of active and fully inactive regimes of large-scale circulation. They rather involve regional changes in convection sites. This is the case in LOVECLIM, and it should finally be noted that the phenomenon has been suggested as a plausible explanation to the existence of Dansgaard-Oeschger oscillations (Vettoretti and Peltier 2015). Our main result is that the introduction of a stochastic parameterization had the ability to *suppress* the oscillation in LOVECLIM (Figure 6).

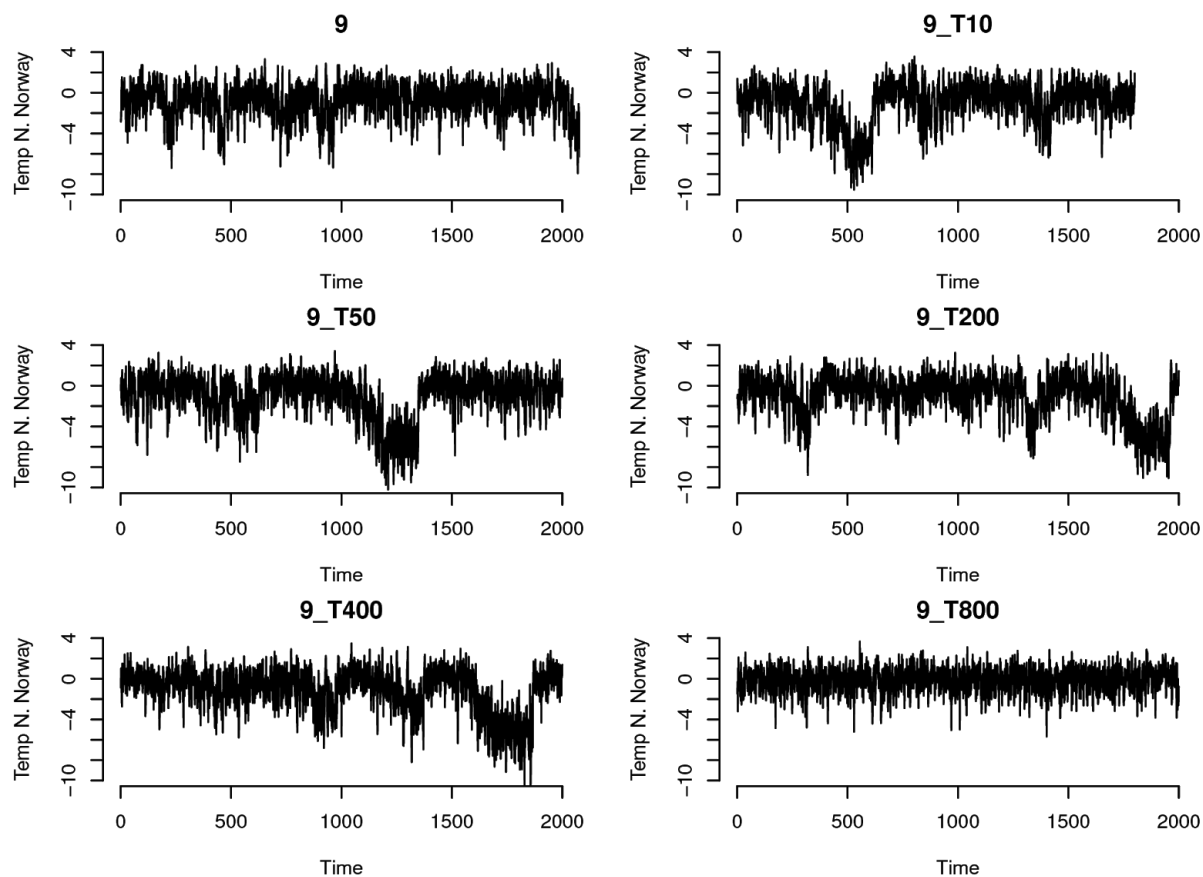


Figure 6: Evolution of the intermittent regime with increasing amplitudes of stochastic parameterizations

The suppression of deep-decoupling oscillations in LOVECLIM was interpreted by reference to two simple models previously published in the literature. On the one hand, the Welander model (Welander 1982) describes the possible existence of a limit-cycle involving dynamics of the ocean water column (convective instability); the Winton and Sarachik (1993) model describes the possibly existence of a limit cycle of the thermohaline circulation coupled with convection in the northern box.

In both conceptual models, we could determine why the introduction of a stochastic parameterization effectively locks the system into its active state. The demonstration relies on a mathematical analysis of the equations. The method of fast-scale averaging allowed us to transform the stochastic dynamical system into a deterministic dynamical system, which could then be analysed classically. Intuitively, one can simply see that the introduction of stochastic parameterization effectively inhibits stratification that causes the development of the off-phase. More details are provided in the scientific report attached as Annex.

In a comprehensive study, Juricke, Palmer, and Zanna (2017) considered three different stochastic schemes, different to ours: stochastic perturbation of the Gent-McWilliams coefficients, stochastic perturbations to the turbulent kinetic energy vertical mixing scheme, and stochastic perturbations to the enhanced vertical diffusion scheme. The latter of the three schemes is closest in spirit to the scheme implemented here. Consistently to our result, the

stochastic perturbation of the enhanced vertical scheme turned out to be the least efficient in enhancing the climate interdecadal variability. Acting on the Gent-McWilliams diffusion coefficient turned out to have, in that study, far greater impact on the interannual variability. Intuitively, this is reasonable. Stability in low-resolution model is artificially enforced by excessive vertical diffusion: The flow is then too viscous to generate enough interdecadal variability. A stochastic scheme targeting the vertical diffusion coefficient of gravitationally stable flows can generate bursts of mixing with a positive effect on low-frequency variability. Our work is however providing an interesting perspective, by showing that stochastic parameterizations may also have a suppressing effects of a form of ultra-low frequency variability that is associated with a slow-manifold attractor (as captured by box-models). Both results are not incompatible and it could have important implications for our understanding and modelling of ocean-climate-ice sheet interactions. In particular, the non-trivial role played by the stochastic forcing in the development of suppression of LFV, as also illustrated in Section 4.2.

4.4 Bayesian approach to calibration

In Bayesian statistics, the calibration of a model may be formulated as the problem of estimating the *posterior* distribution of a model parameter θ , given observations z . This posterior distribution is thus proportional to the product of a *likelihood* and a *prior*.

The key difficulty relates to the fact that the likelihood is generally not available in closed form. It must therefore be estimated by some numerical method, which may either rely on a deterministic estimator (generally biased), or a Monte-Carlo estimator (unbiased, but only true asymptotically). The nature of the problem differs depending on the kind of model that is being calibrated (low-order dynamical system, or GCM), and the objective of the calibration procedure (model selection, identification of process, or prediction). For STOCHCLIM, we considered several applications: (1) calibration of low-order stochastic dynamical systems on observations to identify the presence of a forcing (2) choice of calibration metric in for prediction, and (3) calibration of a computationally more expensive model with a meta-model.

Application (1) yielded several articles in the refereed literature. Applications (2) and (3) require more work before reaching publication standards, but allowed us to formulate recommendations.

Calibration of simple models for Dansgaard-Oeschger events

The scientific rationale starts from the observation that the last glacial period was punctuated by a series of abrupt climate shifts, the so-called Dansgaard-Oeschger (DO) events. The frequency of DO events varied in time, supposedly because of changes in background climate conditions. We therefore used the calibrated models to investigate the influences of external forcings on DO events. Specifically, we assumed two types of generic stochastic dynamical systems models (double-well potential-type and oscillator-type), forced by the northern hemisphere summer insolation change and/or the global ice volume change.

We showed, in Mitsui et al. (2017), that the stochastic oscillator model with at least the ice volume forcing reproduces well the sample autocorrelation function of the record and the frequency changes of warming transitions in the last glacial period across Marine Isotopic Stages 2, 3, and 4 (last 200,000 years) (Figure 7). The study is relevant for the general objective of learning how to calibrate climate models. Indeed, even though the model is very basic and low dimensional, it turned out that the calibrated model performed particularly well by reference to different criteria: it reproduces the power spectrum, the frequency of events, and the generalised Hurst exponents satisfactorily (see Mitsui, Lenoir, and Crucifix (2018)). The latter argument is significant because it allowed us to demystify the origin of the Hurst exponent measured in the time series. There is no need for ‘cascade processes’ or even ‘long-memory’ stochastic processes to explain the non-linear statistics of the observations. Our simple model convincingly reproduces the different *statistics* of the data.

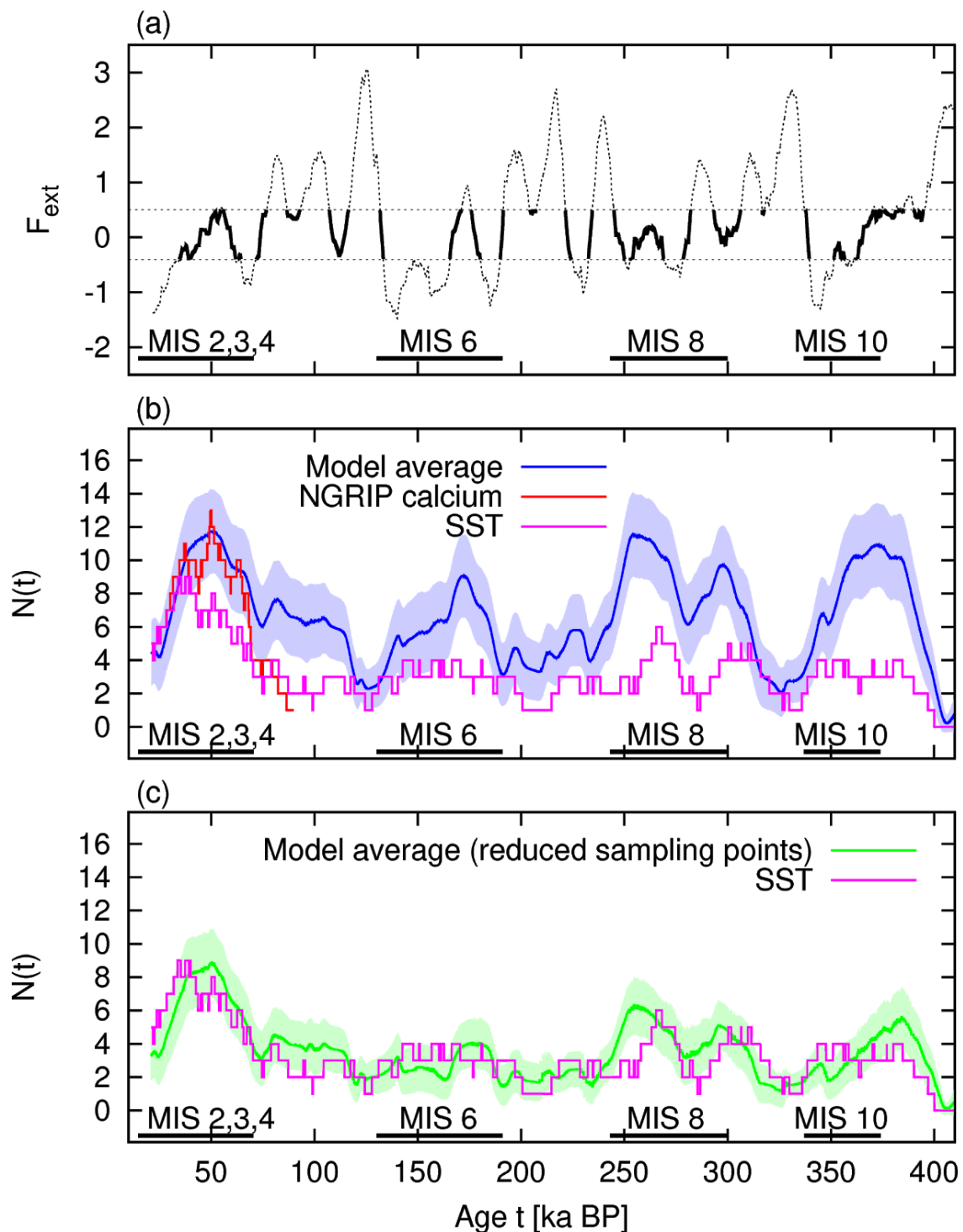


Figure 7: Simulations of abrupt millennial-scale climate changes in the past four glacial periods. (a) Variations of the external forcing. Thick parts of the line highlight the range with active stochastic oscillations (b) The number of warming transitions $n(t)$ over a 20-ka time interval. The red curve is $n(t)$ for the 20-year average NGRIP Ca data. The magenta line is $n(t)$ for the Iberian margin reconstructed SST, and the blue curve is the simulated ensemble mean. The shaded error bar represents one standard deviation. (c) The green curve is the ensemble mean for the simulated time series whose values are sampled at the time points same as the SST recorded.

Sensitivity of calibration on the choice of calibration metric for prediction with an imperfect model

We also considered a simple dynamical system, three-dimensional, known to exhibit a limit cycle. This model was published by Saltzman and Maasch (1990) as a model for ice ages. It is *non-autonomous*, because it includes an influence of the astronomical forcing.

The first step of the procedure is fairly straightforward: one trajectory is simulated with the model (assuming $\sigma = 0.7$). Then a timeseries is constructed using only the observable χ , noised-up (with a noise standard deviation of 0.1), and this data is used to estimate the state of the model, using the correct parameters.

The problem is formulated as a parameter-state estimate of time series. In this kind of problem, the particle filter provides an unbiased estimator of the likelihood, as shown by Andrieu, Doucet, and Holenstein (2010). As we could verify (not shown here, but see the second scientific report put in Annex), in a perfect model experiment, maximising the likelihood provides roughly the same calibration result as maximising predictive accuracy (small differences were attributed to sampling variability, see the remarks about the proper scoring rule in the scientific report).

The procedure is then repeated but assuming that the model generating the dataset and the model used for the calibration are different. We found in that particular case that the likelihood was a very bad metric of predictive accuracy, especially for long term prediction (see the Annex). This result is in fact fairly easily explained. The calibrating model differs from the generating model, while the Bayesian calibration process uses the hypothesis that they are the same. Consequently, the stochastic parameterisation included in the calibrating model must absorb all the discrepancies between the two models. The calibration therefore results in a high noise amplitude, and somewhat distorted physical parameters to compensate for the effect of noise on the invariant measure. The calibrated model is therefore physically inconsistent, and inadequate for predictions.

This result therefore appears as a health warning against likelihood-based model calibration and model selection (e.g.: Carrassi et al. (2017)). A careful characterization of model discrepancy is always essential.

Sensitivity analysis of the VDDG model

We considered now the ocean-atmosphere model presented by Vannitsem (2015). The published version of this model includes 36 ordinary differential equations for the dynamics and thermodynamics of the ocean-atmosphere system. The state vector consequently includes 36 components.

As a first step we illustrated how meta-modelling works as a methodological approach for calibrating a computationally expensive model. To this end, we considered the 36-variable models as such, and assumed that nine parameters are adjustable in the calibration procedure. Physically plausible ranges for all parameters were established with the help of the author of the model, and the resulting parameter space was sampled with a Latin hypercube design chosen following optimality criteria. Every model run predicts a stationary mean and a variance, so that the relationship between the model parameters and the outputs could be mapped continuously. The mathematical object used for mapping inputs onto outputs is a statistical model called a Gaussian process. In statistical term, the Gaussian process is a model of the VDDG *simulator*, and this kind of model is variously called as an *emulator*, a *meta-model*, or a *surrogate*. Given that predictions with this surrogate are computationally cheap, the surrogate may easily be used to solve the problem of calibration.

At that point, the *meta-modelling* strategy consisted in calibrating a stochastic version of the 36-dim model on the deterministic 56-model (following the procedure planned in the research contract). It however turned out that the invariant measures of the two models are so different—the higher-resolution model does not feature the low-frequency oceanic model found in the low-resolution model—that the development of the stochastic parameterization had to be thought out again from first principles (cf. Demayer and Vannitsem, 2017, 2018, and Section 4.2).

4.5 Dynamics and variability under time-dependent forcings: Impact of slow parameter variations

General theory of the impact of slow parameter variations

The possibility to derive universal expressions for the response of the atmosphere and climate to external disturbances, encompassing large classes of systems, is a matter of obvious interest. We have derived a Fokker-Planck equation for the probability distribution of the state variables incorporating the effect of external perturbations, under the assumption that the sources of intrinsic variability can be assimilated to Gaussian random noises. The solution of this equation to the first order with respect to the intensity of the perturbation leads to expressions in the form of time cross-correlations linking the perturbation-induced shift of an observable to the dynamical and statistical properties of the reference (perturbation-free) system. These expressions are reminiscent of those displayed in the classical fluctuation-dissipation theorem derived in Statistical Mechanics, account fully for the nonlinearities present and go beyond the Gaussian approximation usually adopted in the literature.

Representative classes of low-order atmospheric and climate models subjected to Gaussian Markov noise source terms were considered which, as argued by many authors (see e.g. Imkeller and von Storch 2001), capture essential aspects of the intrinsic variability of observables of interest in atmospheric and climate dynamics by accounting, in particular, for the effect of subgrid scale phenomena. We have derived systematically general expressions for their response to weak external forcings in a way that fully accounts for the nonlinearities

present in the intrinsic dynamics (Nicolis and Nicolis 2015). We have shown that the response can be cast into the cross-correlation of the vector of observables of interest and the gradient of a generalized potential function, which also determines the structure of the (generally non-Gaussian) probability density of the reference system. We have derived a nonlinear differential equation satisfied by this quantity and sketch how for any given system this equation can be solved to any desired order by power series expansion. A class of dynamical systems referred to as potential (or alternatively integrable) systems, for which the generalised potential generates not only the invariant probability density but also the time evolution of the state variables in absence of noise has been considered. Explicit forms of the response functions are obtained, in which the role of the nonlinearities and of the strength of the noise can be assessed. In particular, the limitations of the Gaussian approximation in systems possessing long-range correlations are brought out. More general classes of systems are also considered, including systems giving rise to instabilities leading to periodic behaviour. Linearizing the system's dynamics in absence of the forcing around a reference state leads to a Gaussian form of the invariant probability density, whose parameters can be related to those of its intrinsic dynamics. A systematic way to account for nonlinear effects and for non-Gaussian probability densities was also indicated.

The mechanisms by which the climatic system can amplify weak external signals such as the solar influx cycle is a matter of obvious concern in paleoclimatology, notably in connection with the Quaternary glaciations. Stochastic resonance was originally proposed by C. Nicolis as a mechanism for the enhancement of transitions between two coexisting climatic states induced by the presence of noise. The theory elaborated in this original context has been extended to account for complex transition schemes, in which the system switches between a 2-state and a 3-state dynamics upon variation of a suitable bifurcation parameter (Nicolis and Nicolis 2017a). The amplitude of the response to the forcing has been evaluated using linear response theory for a class of stochastic low-order models and shown to display optima for appropriate values of the bifurcation parameter, the forcing frequency and the noise strength.

Stochastic resonance was also recently extended to nonlinear spatially coupled subsystems. Analytic expressions for the different steady-state solutions, for the rates of transitions between them in the presence of noise, and for the response to a weak external periodic forcing have been derived (Nicolis and Nicolis 2017b). It was shown that the presence of spatial degrees of freedom modifies considerably the mechanisms of transitions between states and is responsible for a marked sensitivity of the response on the coupling constant and on the system size.

Analysis of the impact of dynamical noise

We studied the influence of dynamical noise against several models of glacial-interglacial cycles. This analysis outlines a form temporal and quantised instability in systems that are characterised by strange non-chaotic attractors (Crucifix, 2013; and Saltzman and Maasch, 1990). Specifically, the systems are synchronised on the astronomical forcing, but large dispersions of stochastic trajectories can be induced by extremely small noise at key times

when the system is temporarily unstable. After a dispersion event the trajectories remain organised around a small number of clusters, which may co-exist over several glacial cycles until they merge again (Figure 8). The phenomenon is interpreted as a noise-induced excitation of long transient orbits. Dispersion events may be more or less frequent and, depending on the amount of noise, models with strange non-chaotic attractor may have very long horizons of predictability compared to the duration of geological periods.

Compared to this scenario, the model with a smooth attractor (Imbrie and Imbrie, 1980) is always stable, i.e., large dispersion of orbits never occurs. On the other hand, the dynamics of the chaotic model (Hargreaves and Annan, 2002) bare some similarities with the models with strange non-chaotic attractor, in the sense that trajectories cluster, i.e., at a given time the state of the system may confidently be located within a small number of regions. The difference is that there is a larger amount of leakage between the clusters, i.e., individual trajectories escape more easily from the cluster they belong to, and this reduces the predictability of such systems. Finally, we discussed a hybrid dynamical system with a piecewise continuous attractor (Paillard, 1998). Owing to the discontinuity of the attractor, very small amount of noise may rarely induce significant dispersion of trajectories, but contrarily to the scenario with strange non-chaotic attractor, there are no long transients, and trajectories cluster again rapidly (Figure 8). This research, along with other recent works on dynamical systems of ice ages may provide feedback on the design and interpretation of simulations with more sophisticated models.

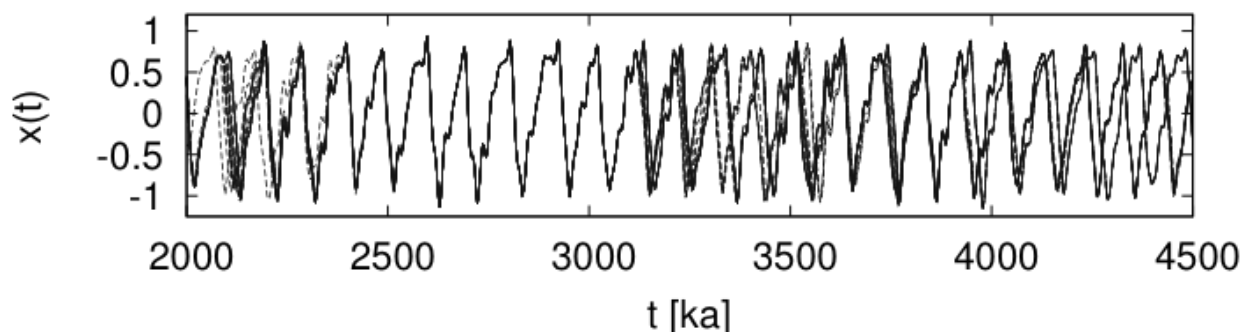


Figure 8: Examples of trajectories generated with a simple oscillator (the ‘Crucifix – DeSaedeleer – Wieczorek’ model) with additive noise, forced by astronomical forcing. Observe that all trajectories are, at certain times, almost identical (between 2500 and 3200 ka), but occasionally split to form so-called long-transients. .

4.6 Dynamics and variability under time-dependent forcings: Precursors of transitions

We have analysed the response of dynamical systems of relevance in atmospheric and climatic dynamics, under the action of time varying parameters emulating external forcings such as anthropogenic effects. We focused on systems giving rise, in the absence of forcing, to transitions between states and showed that the forcing induces a number of unexpected effects. Most prominent among them are that, starting with a stable branch of states, transitions to new regimes that would occur in the “static” case in absence of time variation

of parameters tend to be delayed; states that in the static case are unstable are temporarily stabilized; and states that in the static case are stable can be skipped altogether (Figures 9 and 10). These results were derived by applying a generalized stability criterion extending classical stability analysis to account for the presence of time-varying coefficients in the evolution equations of the system's variables, as well as on analytic solutions prevailing in the vicinity of transition points. They were validated by results of numerical integration of prototypical systems of relevance in atmospheric and climate dynamics giving rise to periodic behaviours to chaotic dynamics and to transitions between simultaneously stable steady states (Nicolis, 2018).

We have identified « forerunner » variables bearing the signature of the transition, of which monitoring would allow one to foresee the transition well before its actual occurrence. Owing to the presence of a large number of variables and parameters and of the uncertainties inherent in the modelling, one could expect that extracting a clear-cut signal of this kind should be a rather difficult task. The strategy that allowed us to achieve our goal has been to appeal to a fundamental result of nonlinear dynamics stipulating that in the vicinity of certain types of transitions the dynamics as described by the full set of equations collapses to a universal form whose structure depends solely on the nature of the bifurcation giving rise to the transition and displays only a limited number of variables. Two types of bifurcation have been analysed in detail (Nicolis and Nicolis 2014): The pitchfork bifurcation, encountered in problems like thermal convection displaying spatial symmetries; and the limit point bifurcation, encountered in problems involving transitions between different modes of atmospheric circulation or between different climatic regimes.

Throughout our approach the time variation of the parameters has been fully and consistently incorporated into the intrinsic time evolution of the system's variables as given by the appropriate rate equations. Our results depend critically on this view of parameter-system co-evolution, a scenario reflecting, we believe, the way a natural system is actually evolving in time. This scenario differs from those adopted in current studies on climatic change based on the integration of large numerical models, where parameters are suddenly set at a different level and the system is subsequently left to relax under these new conditions. It would be interesting to allow for different scenarios beyond the standard ones, closer to our fully dynamical approach, and to test the robustness of the conclusions reached under these different conditions.

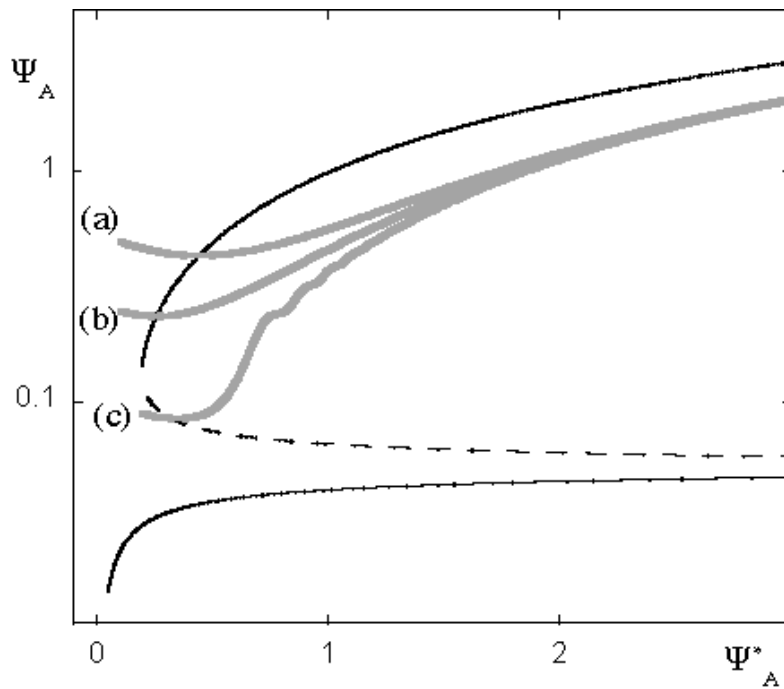


Figure 9: Bifurcation diagram of a low order atmospheric model as a function of parameter Ψ_A^* . Full and dashed lines represent stable and unstable states, respectively. In the presence of a time dependent forcing in the bifurcation parameter initial conditions (a),(b) and (c) evolve to the upper stable branch of the bifurcation diagram although in the absence of the time dependent forcing the system is bound to follow the low stable branch of the bifurcation diagram.

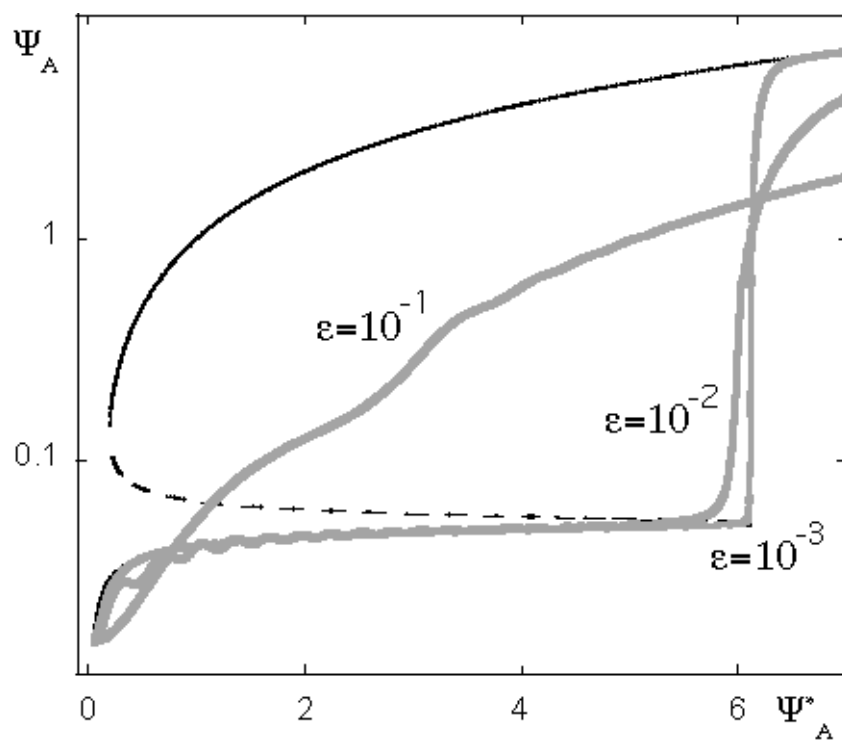


Figure 10: As in Figure 9 but for initial conditions in the vicinity of the lower stable branch and three different amplitudes of the time dependent forcing.

4.7 A stochastic scheme for convection in NWP models

During the STOCHCLIM project a methodology was set-up to apply the theoretical work done by Nicolis (2003) and Nicolis et al. (2009) to the state-of-the-art, high order ALADIN NWP model. More precisely, an investigation in the characteristics of the model error source, related to the absence of a deep convection parameterization scheme was done. Using the same reasoning as Nicolis (2003) the model error, in the absence of initial condition errors, is written as

$$u(t) = (g(t = 0) - f(t = 0))t$$

With u the model error and $g(0) - f(0)$ the model error source. This model error source was obtained by running a twin experiment, where the ALADIN model in a configuration with deep convection parameterization was used as the reference case, while the ALADIN model in a configuration where deep convection was considered explicitly resolved was used the simplified case. Anticipating future work on seasonal forecasting and maximizing the convective cases, the experiment was done over the Tropics during a period of enhanced convective activity.

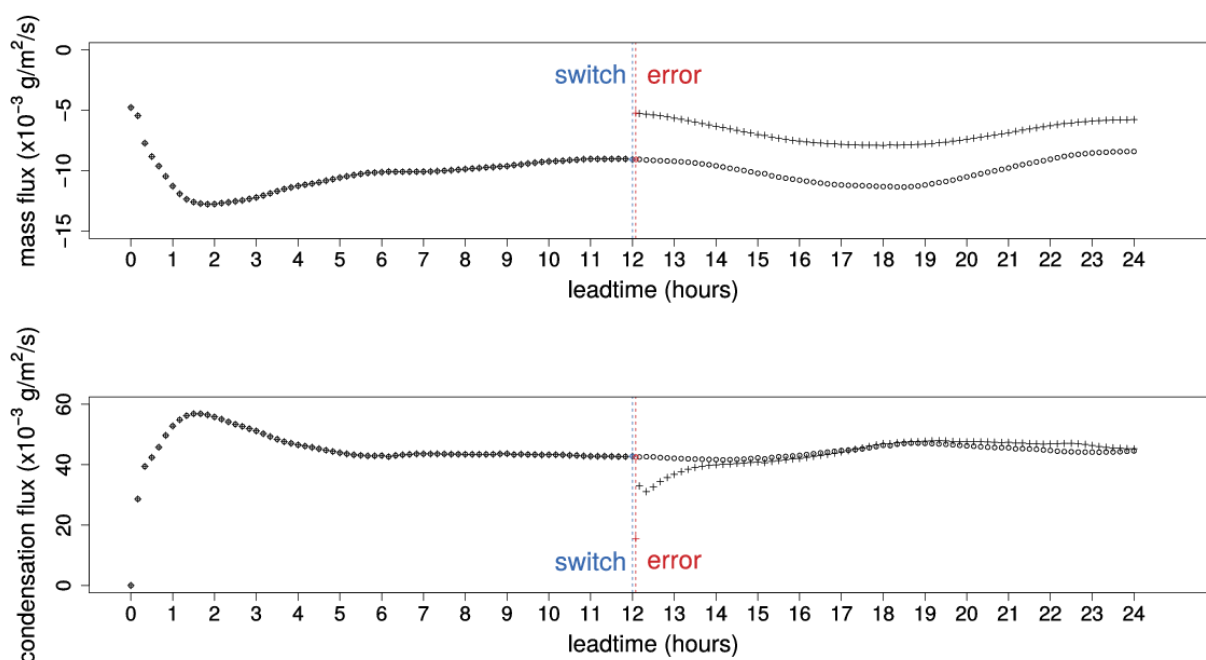


Figure 11. The model error source related to vertical transport (top) and condensation (bottom), expressed as a domain averaged flux.

Figure 11 shows the size of the horizontal and vertical domain averaged model error source for specific water vapor content in the vertical transport and condensation. Investigation of the different time evolution of the model error sources (more or less constant in the case of transport vs. vanishing in the case of condensation) indicates that the main source of error is found in the convective transport.

Figure 12 shows the vertical characteristics of the model error source. The size of the standard deviations is one order of magnitude larger than their respective averaged values, indicating that the deep convection parameterization is responsible for a large amount of variability especially in the middle and upper troposphere. The error source is further characterized by a non-vanishing mean and features related to the absence of both up- and downdraughts.

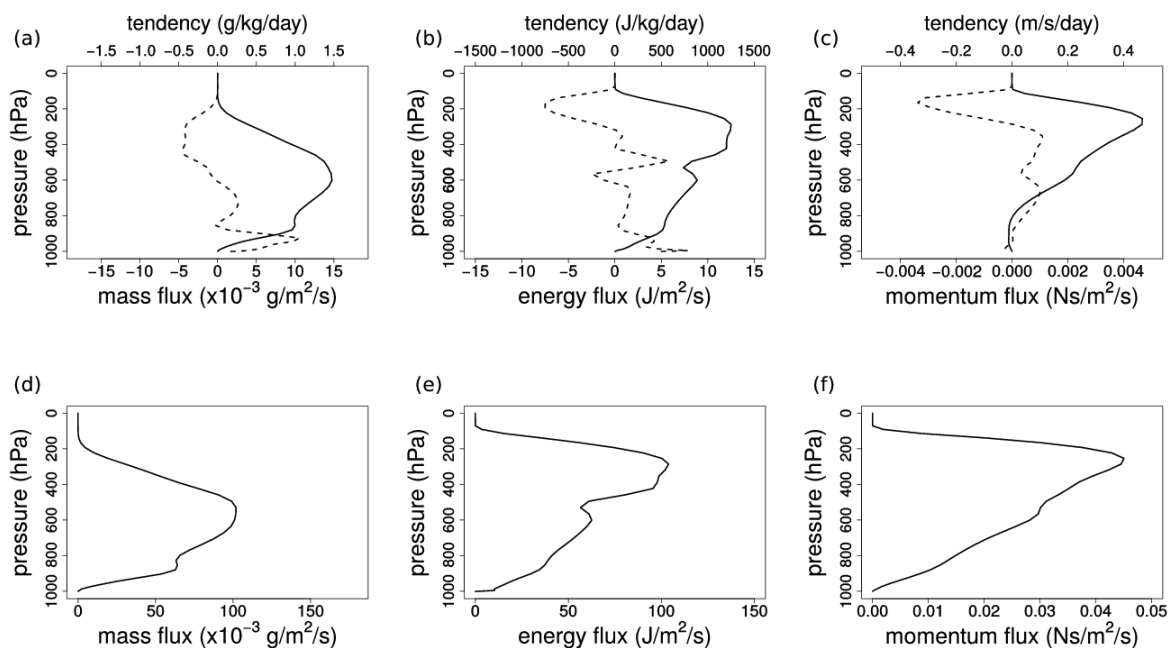


Figure 12. Vertical profile of error source mean flux (solid line) and the associated tendency (dashed line) for water vapor (a), enthalpy (b) and zonal wind (c). The bottom row shows the respective standard deviations.

The probability density function of the error source on one vertical level (Figure 13) reveals a combination of two log-normal distributions. The one centred around zero is coming from grid-points where no convective activity is present, while the other log-normal distribution is coming from grid points with convective activity. These results contain the main statistical properties of the model error source to be used in the stochastic scheme for convection.

Next, a first attempt in building such a scheme was made. The model error source of all the prognostic variables is stored for a representative set of grid points. Subsequently, a model run is performed with deep convection considered explicitly resolved. During the integration the model error source is sampled from this database. The error sources are sampled uniformly and added in those grid points meeting labeled convectively active. For the selection of the active grid points two configurations are tested. One uses the moisture convergence in the planetary boundary layer, while the other uses the vertically averaged resolved vertical velocity. Both selection parameters were found to be good proxies for convective activity.

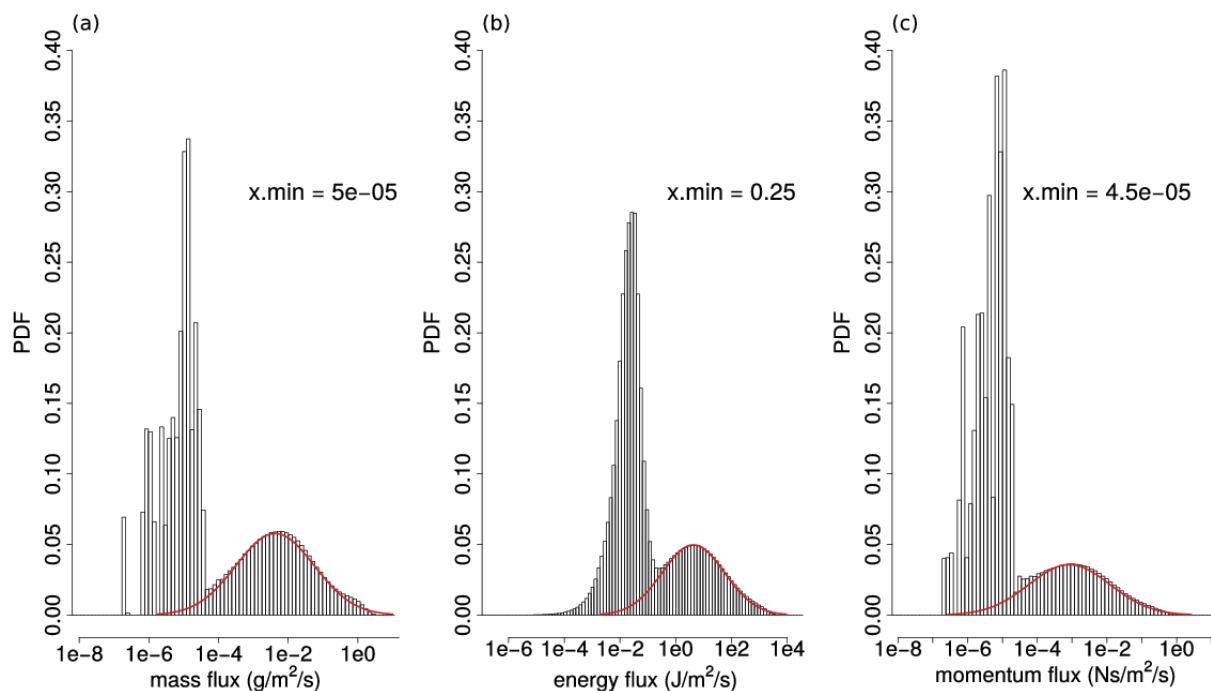


Figure 13. Probability density function at the level of highest variability for water vapour (left), enthalpy (middle) and zonal wind (right). Plotted in red are the fitted log-normal distributions, where only values above x_{min} were used for the fit.

Both the evaluation of the grid point selection criteria as the sampling from the database is done every time step. For now, no spatial or temporal correlation in the error sources is imposed, only the grid point vertical correlation between the error source of the different prognostic variables is preserved.

4.8 Improving forecasting using stochasticity in NWP models

The new stochastic forcing scheme described in the previous chapter was tested during a 10 day period (April 11 – 20, 2009). The ALADIN model configuration with deep convection assumed explicitly resolved (called simplified from here on) and the configuration with deep convection assumed resolved together with the stochastic forcing (called stochastic from here on) were compared relative to the results of the ALADIN reference model configuration with deep convection parameterization scheme. A first test was done comparing the deterministic simplified run with the ensemble mean of a 5 member stochastic ensemble.

The root mean square error (RMSE) of the ensemble mean is significantly reduced when using the stochastic forcing (both configurations) for all prognostic variables both in the lower and middle troposphere except for 500 hPa wind (see Figure 14). This reduction in RMSE becomes apparent after 8 hours and is largest for the configuration using resolved vertical velocity as proxy for convective activity (OMEGA). The ensemble spread induced by the stochastic forcings is underdispersive, reaching about 50 % of the RMSE. Only in the condensates does the stochastic forcing provide a reasonable spread of up to 90% of the RMSE.

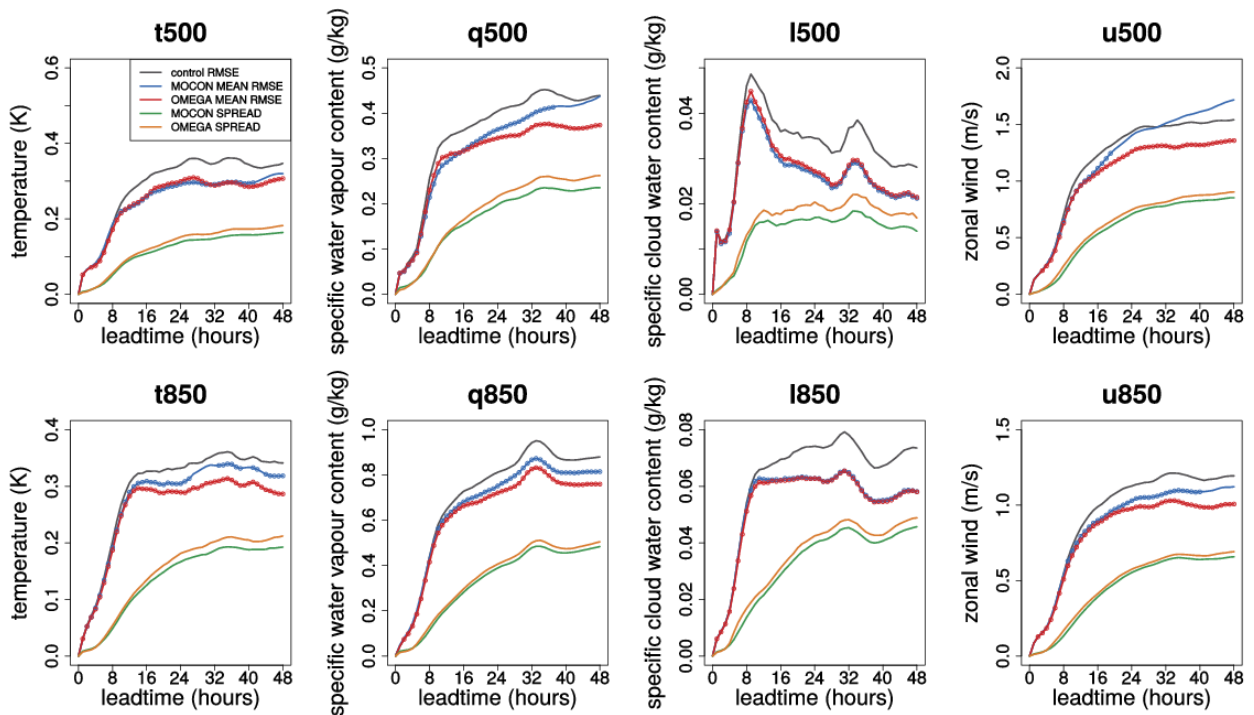


Figure 14. RMSE of the deterministic simplified model run (black), the ensemble mean of the ensemble with stochastic forcing with moisture convergence as proxy for deep convection MOCON (blue) and resolved vertical velocity as deep convection proxy (OMEGA) (red). Ensemble spread is displayed in green for the OMEGA configuration and in orange of the MOCON configuration. In the top row the scores for temperature (t_{500}), water vapour specific content (q_{500}), cloud water specific content (l_{500}) and zonal wind (u_{500}) at 500 hPa are shown. The bottom row shows the same variables but at 850 hPa.

The Continuously Ranked Probability Score (CRPS), the integral of the overall possible Brier scores, offers a measure to compare different distributions and reduces to the Mean Absolute Error for a deterministic forecast. Applying this score to the precipitation distributions (Figure 15), one can see that also here there is an improvement of around 30% with respect to the simplified deterministic run.

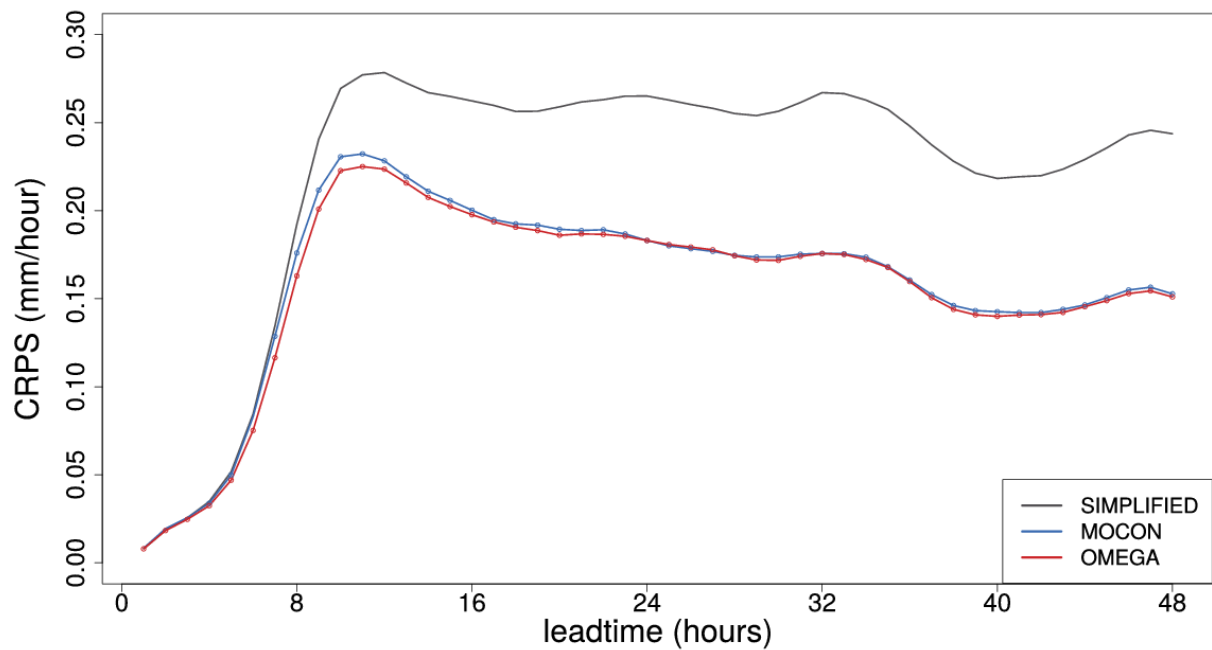


Figure 15. Continuously Ranked Probability score of the precipitation distribution for the deterministic simplified forecast (reduces to mean averaged error) in black. The CRPS for the MOCON and OMEGA configuration are colored blue and red respectively.

Finally a lagged-initial-conditions ensemble running with the simplified configurations was compared with an ensemble combining lagged initial conditions and stochastic forcing. The skill scores summarized in Fig. 6 show that also here there is an improvement in forecast skill. RMSE skill scores for 500 hPa temperature are drastically improved when using the moisture convergence as proxy for deep convection, while the improvements for cloud liquid is more moderate. For water vapor and zonal wind there is only an improvement in the first 16 to 24 hours, after that the RMSE worsens. While the OMEGA configuration of the scheme has a more moderate impact on the RMSE, its impact is positive for all variables and lead times. Both configurations bring also the spread closer to that of the reference ensemble.

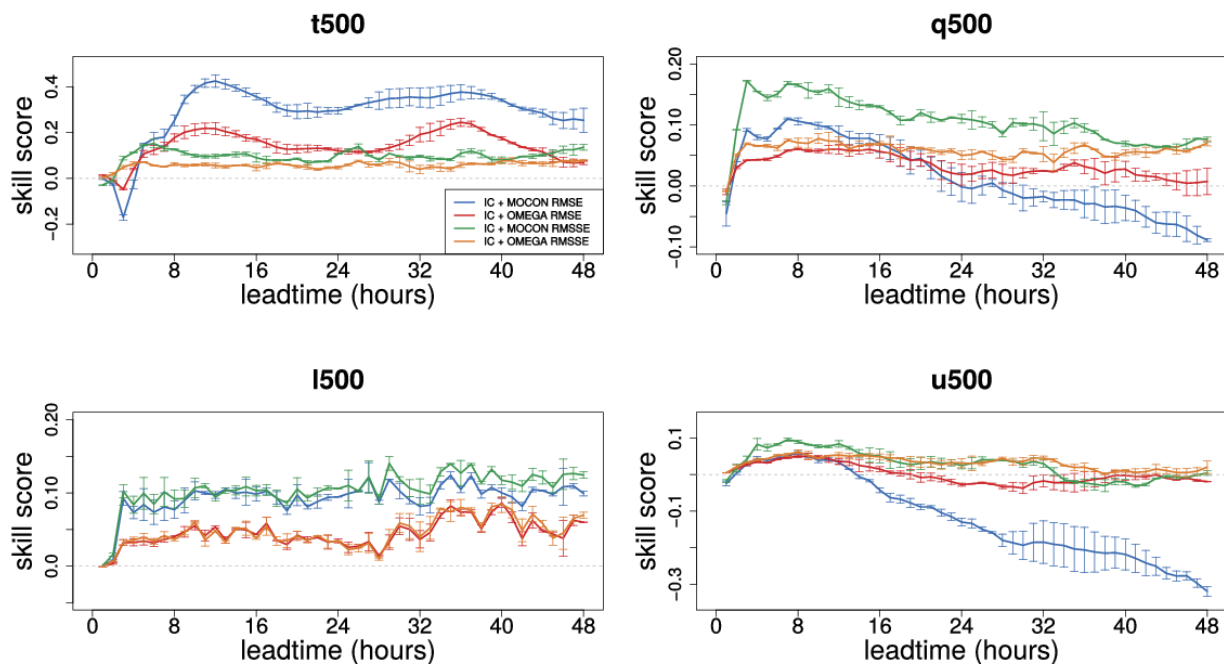


Figure 16. Skill score of the RMSE and the Root Mean Square Spread Error of the ensemble mean and ensemble spread of the MOCON (red, orange) and OMEGA (blue, green) configurations with respect to the simplified ensemble. The naming convention for the different variables is the same as that of Figure 14.

Comparing the precipitation distributions for the lagged-initial-conditions ensembles (not shown) the improvement seems to be moderate, with a reduction in CRPS of around 10%. The results presented above show the potential of the scheme developed during the project. Some improvements, for instance introducing a spatial and/or temporal correlation between the error sources, can definitely still be made. It would also be interesting implementing this scheme in a larger ensemble using perturbed initial conditions coming from a state-of-the-art data assimilation scheme.

4.9 Conclusions and recommendations

The main conclusions and recommendations are formulated by splitting the different topics that were addressed during the course of the project.

- 1. Development of a low-order coupled ocean-atmosphere model.** During the course of the project the necessity of clarifying the role of the ocean and of other slow components of the climate system on the development of the low-frequency variability in the atmosphere emerged. This led us to develop a hierarchy of coupled ocean-atmosphere models, of which the MAOOAM model framework (Modular Arbitrary Order Ocean Atmosphere Model) is the final product. This model is freely available on the portal Github. These developments have allowed us to clarify the emergence of the low-frequency variability within the atmosphere (in the model) associated with the presence of the ocean. A complex dependence of its emergence is

found as a function of the coupling parameters of the model, far from the common believe that once the ocean is coupled to the atmosphere (whatever is the strength of the coupling), the atmosphere displays a low-frequency variability associated with the dynamics of the ocean and hence a higher predictability. This very important finding in our hierarchy of models should now be explored in state-of-the-art climate models.

- 2. Necessity of the development of first principle stochastic modelling.** During the course of the project, several techniques for stochastic modelling of subgrid scale processes derived from first principles, were explored. This was done in a hierarchy of models from low-order $O(10)$ to intermediate order $O(100)$ coupled ocean-atmosphere models, thanks to the development of the MAOOAM model framework. Two main methods were explored that differ by the type of assumptions made on the coupling between the resolved and unresolved (subgrid scale) processes, the first one on the infinite separation of time scales between the resolved and unresolved processes (referred to as MTV in the report, also called homogenisation) and the second one on a weak coupling between them (referred to as WL in the report). Both methods provide good results, but the former method is easier to implement as it does not involve integro-differential equations. The MTV method is certainly a good avenue for future research on stochastic modelling in the context of state-of-the-art climate models.

- 3. Calibration requires a good characterisation of model discrepancy, and a good understanding of the implications of model discrepancy.** As a general rule, calibrating a Gaussian-process model to determine statistically the relationship between the inputs of a numerical model and its outputs is a viable strategy. This is called "meta-modelling" or "emulation". We found it reasonably easy to calibrate a Gaussian processes to model the relationship between model input and output in two different applications: the HadCM3 GCM simulations of Pacific variability (3 parameters) and the 36-variable VDDG model (8 parameters), even though the VDDG model presents a bifurcation structure causing discontinuities in the relationship between inputs and outputs. It would certainly be possible to design better meta-models in this case, but the possible shortcomings of the Gaussian process were not the limiting factor in our case. Indeed, meta-modelling is not helpful to calibrate a model, if the discrepancy between the model and the calibration target is not properly characterized in the first place. We therefore recommend furthering reflecting on the strategies for the representation of model discrepancy. Specifically, we suggest that because of model discrepancy, the most appropriate metric for calibrating a model may differ depending on the objective of the calibration, whether this is prediction at different time horizons, the identification and quantification of forcing agents, or model selection. This conclusion is generally accepted by statisticians, but still under-appreciated in numerical weather prediction and climate research. The literature on the context-dependent calibration of numerical weather prediction models is non-existent and this gap need to be filled.

- 4. Research on low-order models must be further encouraged.** We found in our research on palaeoclimates that small dynamical systems remain useful to consolidate our understanding of climate dynamics. For example, calibrating forced oscillators or potential-well (Langevin)-type stochastic models on palaeoclimate data allowed us to identify the effects of insolation and glacial forcings on the frequency of Dansgaard-Oeschger events. Further analyses of these dynamical systems clarified the origin of the Hurst exponent measured in ice-core data. We therefore recommend to keep promoting the teaching of dynamical systems theory in climate classes. Stochastic, low-order dynamical systems provide a richer set of concepts than linear theory for expressing climate sensitivity and rapid climate change. A pro-active research strategy will necessarily involve some pragmatism (for example, use of numerical simulations and ad hoc algorithms). Nevertheless, we recommend researchers to actively participate to meetings involving mathematicians (dynamical system experts and statisticians). For example, interactions with statistician Richard Wilkinson attracted our attention on the interest of a proper scoring rule for the calibration of climate models.
- 5. Research on centennial variability needs be promoted.** Our attempts at enhancing centennial variability in LOVECLIM with a simple stochastic scheme for convection did not succeed. One plausible explanation is that the current schemes of vertical and horizontal turbulent diffusion in ocean models have been adopted and built for stability, with potentially negative consequences for the representation of the slow modes of motion. Yet, dynamical system modelling and palaeoclimate data analysis suggest that current general circulation models may mis- or under-represent centennial variability, with potential consequences for our attribution of the current temperature trend.
- 6. Development of consistent stochastic schemes for atmospheric convection.** In the context of a state-of-the-art Numerical Weather Prediction model, a new stochastic scheme for deep convection has been proposed based the theory of model error estimation. The scheme is based on perturbations of fluxes of mass, energy and momentum, independent in space and time. The scheme, although quite crude, allowed for important improvements of the quality of the forecasts of the ALADIN model version as compared to the reference run (in which deep convection is explicitly deterministically parameterized, the ALARO model version). This very encouraging result calls for further research on the development of deep convection stochastic schemes, with modifications of the assumptions made on the spatio-temporal properties of the stochastic forcing and the incorporation of moisture fluxes perturbations.
- 7. Impact of slow variations of external forcings.** The problem of the impact of slow variations of parameters (non-autonomous systems) is a very important topic in the context of climate projections and predictions. This problem has been addressed through the theoretical investigation of the impact of slow forcings in a series of

prototypical models of the atmospheric and climate dynamics. Forerunner variables have also been identified bearing the signature of the transitions, whose monitoring would allow one to foresee transitions well before its actual occurrence. These analyses should now be extended to state-of-the-art climate models in order to clarify whether typical transitions can be predicted.

4.10 References

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5. DISSEMINATION AND VALORISATION

Products

The dissemination has been done through publication of the results in international peer-reviewed journals and through presentations at different meetings. The list of publications in which the project is acknowledged, is provided in Section 6.

During the course of the project, the dynamics of the low-order coupled ocean-atmosphere model allowing for incorporating as many modes as desired has been explored for a number of variables from 36 to about 500. This model has been called “MAOOAM”. The model has been made available on both the website of the journal *Geoscientific Model Development* (<http://www.geosci-model-dev.net/9/2793/2016/gmd-9-2793-2016-assets.html>) and on GitHub (<https://github.com/Climdyn/MAOOAM>).

Workshops

Two workshops have been organized during the course of the project, at which most of the members of the follow-up committee were present.

Workshop 1

This workshop, entitled “Advances in Climate Theory”, has been organized on 25-27 August 2014 at the Royal Meteorological Institute, jointly with the Université Catholique de Louvain (M. Crucifix) and University of Hamburg (V. Lucarini). It has been the occasion to meet two members of the follow-up committee. The programme of the workshop is available as an Annex.

Workshop 2

The second workshop has been organized on September, 21, 2017 in order to evaluate the scientific activities and progresses that have been made during the course of the STOCHCLIM project financed by Belspo. The workshop has been organized on one day and the members of the follow-up committee have been invited to participate (Tim Palmer, Jun-ichi Yano and Daan Crommelin were present), as well as the science officer of Belspo in charge of our project, Martine Vanderstraeten. The themes of the workshop have covered two different (complementary) aspects, first the development and evaluation of new techniques for stochastic modelling of subgrid scale (unresolved) processes, and second the practical implementation of stochastic processes for the improvement of forecasts in state-of-the-art models. About 40 people were attending the meeting, mostly from the RMI and people working in the context of the project.

All the results and the discussions concur to indicate that modelling unresolved processes using stochastic forcing are very effective for the improvement of the statistics of the climate

of models, provided it is properly designed. In state-of-the-art models such stochastic forcing is usually very crude, but it nevertheless allows for improving certain aspects of their natural variability and also their ability to improve the reliability of ensemble forecasts. The STOCHLIM project contributes successfully to that program by testing new techniques in low and intermediate order climate models, and in a state-of-the-art regional numerical weather prediction model.

Yet there is a gap between the success of the new theoretical approaches that were recently developed and their operational implementation in state-of-the-art models. Bridging this gap is challenging and should be pursued in the future for getting climate models displaying the right variability at all frequencies. The program of the workshop is provided below,

Stochastic modelling of subgrid scale processes: From theory to practice

Date: September 21, 2017

Place: Royal Meteorological Institute of Belgium, Av Circulaire, 3, 1180 Brussels, Belgium

9:00 Opening

9:20 Takahito Mitsui and Michel Crucifix, Title: Characterization of the stochastic dynamics of the glacial climate based on the Greenland records ([UCL](#))

10:00 Jonathan Demaeyer and Stéphane Vannitsem, Title: Analysis of recent physically-based approaches for stochastic modelling of subgrid scale processes ([RMI](#))

10:40 coffee

11:00 Valerio Lucarini, Title: Statistical Mechanics of Geophysical Flows: Response and Parametrizations ([University of Reading, UK](#))

11:40 Jeroen Wouters, Title: Edgeworth expansions as a method for parameterizing multiscale systems ([University of Reading, UK](#))

12:20 Daan Crommelin, Title: Data-driven methods for stochastic parameterization ([CWI, The Netherlands](#))

13:00 lunch

14:00 Christian Franzke. Title: Energy conserving stochastic models of the atmosphere ([University of Hamburg, Germany](#))

14:40 Michiel Van Genderachter and Piet Termonia. Title: Feasibility study of a model-error based sampling method for stochastic perturbations for NWP models ([Gent University](#))

15:20 coffee

15:40 Tim Palmer. Title: Nonlinear model error and the overspreading of seasonal forecast ensembles ([University of Oxford, UK](#))

16:20 Jun-Ichi Yano. Title: the sudden onset of convection without trigger: probabilistic description without stochasticity ([Météo-France, France](#))

Outreach activities

M. Crucifix co-edited with C. Franzke and A. De Vernal a special issue of the journal “PAGES Magazine”, targeted at scientists and scientific executives, summarizing the state-of-the-art about centennial variability. Several project results were featured in that special issue. The special issue is available on line at: <http://pastglobalchanges.org/products/pages-magazine/11504-25-3-centennial-millennial-clim-var>

M. Crucifix and S. Vannitsem have become part of the scientific committee of the PAGES “Centennial Variability Across Scales” working group. (<http://www.pages-igbp.org/ini/wg/intro/143-initiatives/working-group/dice/1369-cvas-climate-variability-across-scales>)

M. Crucifix co/presented two lessons at the Collège Belgique (October 2016) were work delivered under the STOCHCLIM contract was advertised and shown.

S. Vannitsem has published a paper in the journal “La Météorologie” whose purpose was to summarize results on the use of low-order models in the understanding of the development of low-frequency variability on atmospheric and climate models for the general public (in French). The full reference is: S. Vannitsem, Que nous apprennent les modèles météorologiques et climatiques simplifiés sur la prévisibilité à long terme de l’atmosphère? accepted in *La Météorologie*, 2018.

6. PUBLICATIONS

Peer-reviewed publications linked with the project

1. M. Crucifix, T. Mitsui, G. Lenoir (2016) , Challenges for ice age dynamics: a dynamical systems perspective, *Nonlinear and Stochastic Climate Dynamics* (eds: C. Franzke , T. J. O’Kane) 1-32 [doi:10.1017/9781316339251.002](https://doi.org/10.1017/9781316339251.002) [arxiv:1512.03557](https://arxiv.org/abs/1512.03557)
- 2.
3. De Cruz, L., J. Demaeyer and S. Vannitsem, 2016. The Modular Arbitrary-Order Ocean-Atmosphere Model : MAOOAM v1.0, *Geoscientific Model Development*, **9**, 2793-2808.
4. De Cruz, L., Schubert, S., Demaeyer, J., Lucarini, V. and Vannitsem, S., 2018. Exploring the Lyapunov instability properties of high-dimensional atmospheric and climate models, *Nonlin. Proc. Geophys.*, **25**, 387-412.
5. Demaeyer, J., and Vannitsem, S., 2017. Stochastic parametrization of subgrid-scale processes in coupled ocean–atmosphere systems: benefits and limitations of response theory. *Quart. J. Royal Met. Soc.*, **143**(703), 881-896.
6. Demaeyer, J., and Vannitsem, S., 2018. Stochastic Parameterization of Subgrid-Scale Processes: A Review of Recent Physically Based Approaches. In *Advances in Nonlinear Geosciences* (pp. 55-85). Springer.
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8. G. Lenoir, M. Crucifix (2018) , A general theory on frequency and time--frequency analysis of irregularly sampled time series based on projection methods -- Part~1: Frequency analysis, *Nonlin. Proc. Geophys.*, **25** 145--173 [doi:10.5194/npg-25-145-2018](https://doi.org/10.5194/npg-25-145-2018)
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10. Mitsui, T., M. Crucifix, K. Aihara, 2015. Bifurcations and strange nonchaotic attractors in a phase oscillator model of glacial--interglacial cycles, *Physica D: Nonlinear Phenomena* **306** 25 - 33 [doi:10.1016/j.physd.2015.05.007](https://doi.org/10.1016/j.physd.2015.05.007) (arxiv available at [arxiv:1506.04628](https://arxiv.org/abs/1506.04628)).
11. Mitsui, T. and M. Crucifix, 2016. Effects of Additive Noise on the Stability of Glacial Cycles, *Mathematical Paradigms of Climate Science* (eds: F. Ancona, P. Cannarsa, C. Jones, A. Portaluri) 93--113 [doi:10.1007/978-3-319-39092-5_6](https://doi.org/10.1007/978-3-319-39092-5_6) [arxiv:1611.03295](https://arxiv.org/abs/1611.03295).
12. Mitsui, T. and M. Crucifix, 2017. Influence of external forcings on abrupt millennial-scale climate changes: a statistical modelling study, *Climate Dynamics* **48** 2729-2749 [doi:10.1007/s00382-016-3235-z](https://doi.org/10.1007/s00382-016-3235-z)
13. Mitsui, T. G. Lenoir, and M. Crucifix, Is glacial climate scale invariant ? Submitted to “Dynamics and Statistics of the Climate System”

14. Nicolis, C. and G. Nicolis, 2014. Dynamical responses to time-dependent control parameters in the presence of noise: A normal form approach, *Phys. Rev. E* **89**, 022903.
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21. Nicolis C. and G. Nicolis, 2017. Coupling-enhanced stochastic resonance *Phys. Rev. E* **96**, 042214.
22. Nicolis C. and G. Nicolis, 2017. Stochastic resonance across bifurcation cascades, *Phys. Rev. E* **95**, 032219.
23. Nicolis, C. 2018. Climatic responses to systematic time variations of parameters: A dynamical approach, *Nonlin. Proc. Geophys.*, **25**, 649-658.
24. P. C. Tzedakis, M. Crucifix, T. Mitsui, E. W. Wolff (2017) , A simple rule to determine which insolation cycles lead to interglacials, *Nature* **542** 427–432 [doi:10.1038/nature21364](https://doi.org/10.1038/nature21364) (side product of the project)
25. Van Genderachter M., D Degrauwe, S. Vannitsem and P. Termonia, 2018. Towards model-error based stochastic perturbations in convection-aware ensemble prediction systems, submitted to *Mon Wea Rev*, May 2018.
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8. ACRONYMS

ALADIN	Aire Limitée Adaptation Dynamique Développement International
ALARO	Aire Limitee Adaptation/Application de la Recherche a l'Operationnel
AROME	Applications of Research to Operations at MEscale
CRPS	Continuous Rank Probability Score
DO	Dansgaard-Oeschger
ECMWF	European Centre for Medium-range Weather Forecasts
ENSO	El-Nino-Southern-Oscillation
HARMONIE	HIRLAM–ALADIN Research on Mesoscale Operational NWP in Euromed
GCM	General Circulation Models
IPCC	Intergovernmental Panel on Climate Change
IRMB	Institut Royal Météorologique de Belgique
LFV	Low-Frequency Variability
LOVECLIM	Loch-Vecode-Ecbilt-Clio-aglsm Model
MAOOAM	Modular Arbitrary-Order Ocean-Atmosphere Model
MJO	Madden-Julian Oscillation
MOCON	MOisture CONvergence
MTV	Majda-Timofeiev-Van den Einden
NAO	North Atlantic Oscillation
NGRIP	North GREENland Ice core Project
NWP	Numerical Weather Prediction
QBO	Quasi-Biennial Oscillation
RMI	Royal Meteorological Institute
RMSE	Root Mean Square Error
UCL	Université Catholique de Louvain
U Gent	Universiteit Gent
WL	Wouters - Lucarini

9. ANNEXES

3 annexes are provided:

1. On the problem of model calibration, entitled “Calibration of stochastic parameters by Bayesian methods”
2. Internal report on the use of stochastic forcing in LOVECLIM, entitled “Stochastic parameterization of deep-ocean convection in LOVECLIM”
3. The programme of the Workshop held in August 2014 at the Royal Meteorological Institute of Belgium.