Quasi-geoid BG03 computation in Belgium

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Introduction

The estimate of a high precision quasi-geoid is nowadays a relevant goal in Geodesy, since from this surface can be derived the geoid. As it is well known, the geoid, i.e. the equipotential surface of the Earth gravity field which is close to the mean ocean surface, can be used in combination with GPS observations to estimate orthometric heights. This is of particular relevance, since this can be done in a faster and cheaper way than using spirit leveling, although with lower precision (which is however sufficient in many practical applications). In 1996, the last estimate of the Belgium quasi-geoid BG96 was computed with the Stokes and the least square collocation methods (Pâquet et al 1997). This quasi-geoid has a precision of 3 to 4 cm in the area well covered by gravity data , which was assessed through comparison with GPS/leveling derived undulations with 36 BEREF points. Since now the gravity coverage of Belgium is completed a higher precision for the geoid could be reached for the south-eastern part and in the northern part of the country.

In this paper, a new estimate of the Belgium quasi-geoid (BG03) is presented. The main improvements with respect to the previous computation are related to gravity data coverage, DTM refinements and new global geopotential models. So, this estimate can be considered a significant step forward in quasi-geoid computation for this area and a basis for a future estimate which will be obtained by merging gravity and GPS/leveling data.

1. Gravity data, DTM and global geopotential models

The gravity date base used in this computation has been sharply improved with respect to the previous one. Furthermore, a new DTM has been prepared including bathymetry. The details related to these new data sets will be discussed in the following together with a description of the geopotential models used to represent the low frequency part of the geopotential field.

1.1 The gravity data set

A first determination of the gravity value in Uccle (ROB) was obtained in 1894 with the help of a pendulum. A first belgian gravity Network with 24 stations was successfully observed in 1928 with an internal error ranging from 1 to 3 mGal. In the years 1947-48 a second gravity survey of the country was performed including 381 stations to cover a territory of 30 000km^2. The precision was everywhere better than 0.7 mGal. Since 1948 The National Geographic Institute (NGI) and the Royal Observatory (ROB) worked in close co-operation to densify the gravity coverage of Belgium. This goal was finally reached in 2002. The density of the coverage is lower in the south-eastern part of the country (1 station per 2.5 km^2 to 1 per 5 km^2) but it reaches 1 station per km^2 on the rest of the territory. The data base of the ROB holds more than 250 000 gravity measurements for Belgium and the surrounding countries. All these gravity values were include in BG03. The precision is everywhere better than 0.1 mGal. There are more than 30 000 data on the Belgian territory itself. The rest of the data were provided by the BRGM for France, the BGS for Great Britain, the Rijkswaterstaat for the Netherlands, and Wenzel H.G. (personal communication) for Germany. All those data have been carefully validated. All networks are referenced to the gravity datum of Uccle 1976, (Poitevin 1980).

1.2 The Digital Terrain Model

In the framework of this computation, a new DTM has been set up to properly compute the terrain effect. In Belgium the DTM has been provided by the NGI with a resolution of 3'×6'. For the surrounding territories in the window

$$38° \leq \varphi \leq 54° \qquad -6° \leq \lambda \leq 13°$$

an homogeneous 4 km grid was obtained by integrating the land data of the WEEG Project (Fairhead, 1994) with the 5' NOAA bathymetry in the same area.
The DTMs were merged using bilinear interpolation to produce a unique DTM with spacing $\Delta\varphi$=2.5' and $\Delta\lambda$=3' and boundaries

$$47.5° \leq \varphi \leq 53.5° \qquad 0° \leq \lambda \leq 8°$$

In this way, the estimated DTM is known over an area that is one degree larger than the one corresponding to the gravity data.
The plot of this DTM is shown in fig.1

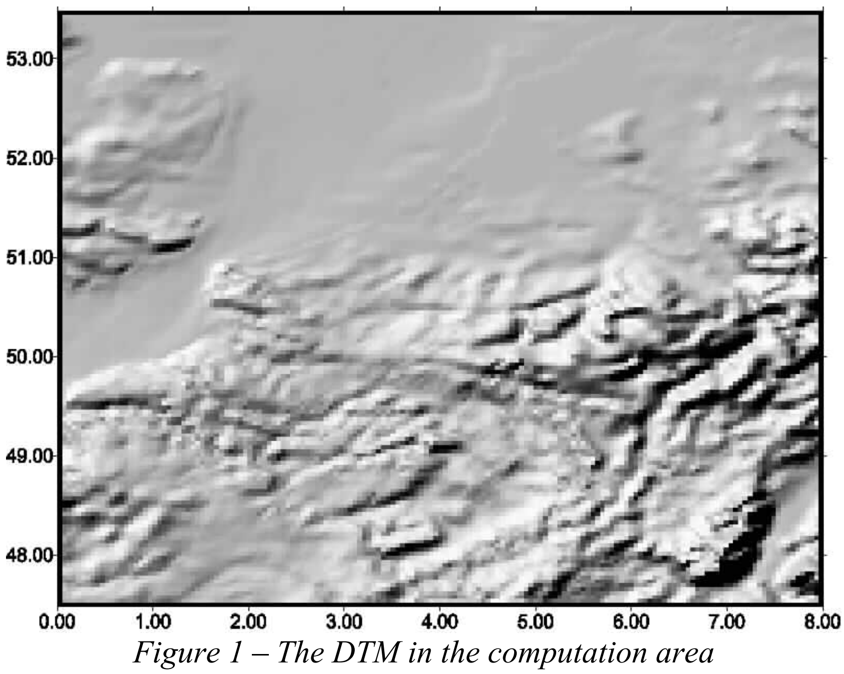

Figure 1 – The DTM in the computation area

1.3 The global geopotential models

Since the previous estimate of the quasi-geoid in Belgium, which was based on OSU91A, two new geopotential models have been made available: EGM96, complete up to degree 360, (Lemoine et al, 1998; IGeS Bulletin, 1997) and the high resolution model GPM98CR by Wenzel, complete up to degree 720, (Wenzel, 1998). The plots of the gravity anomaly implied by the two more recent models and by OSU91A, bounded to the computation area, are shown in fig 2.

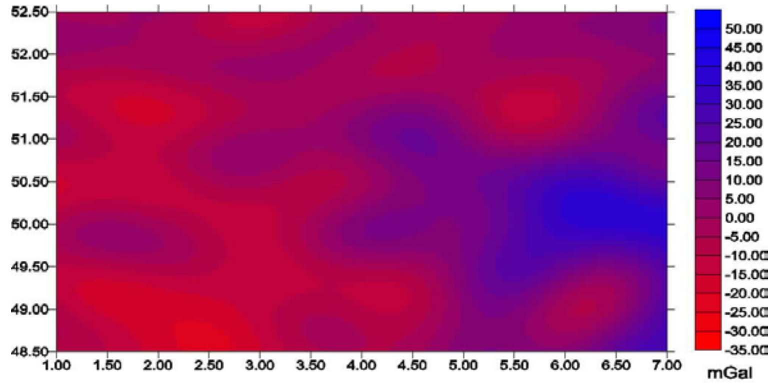

Figure 2a - The OSU91A gravity anomaly

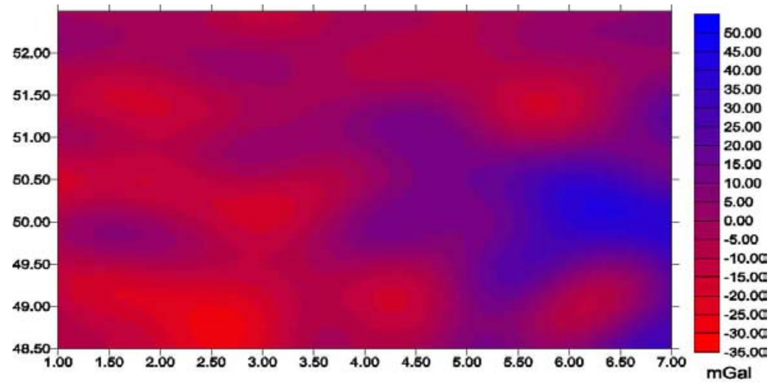

Figure 2b - The EGM96 Gravity anomaly

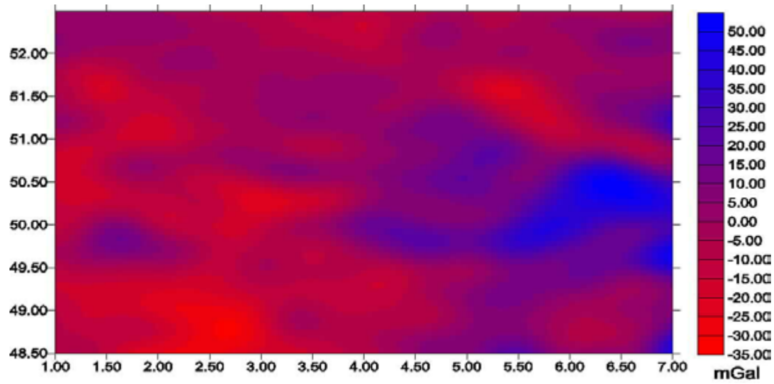

Figure 2c - The GPM98CR gravity anomaly

As one can see, Δg(OSU91A) and Δg(EGM96) are quite similar while Δg(GPM98CR) displays a rougher structure. The same consideration holds for the model undulations which are plotted in fig 3.

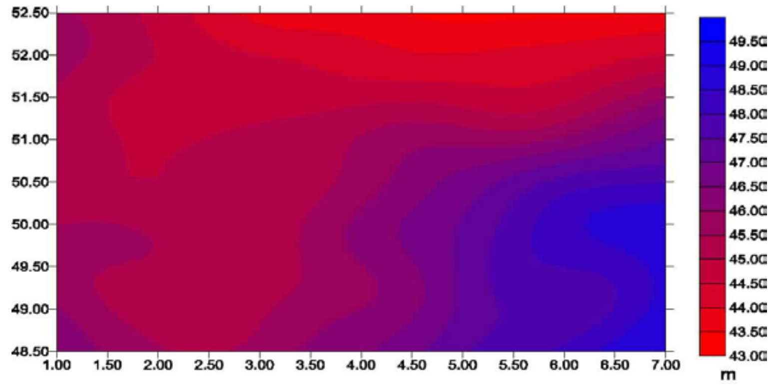

Figure 3a - The OSU91A undulation

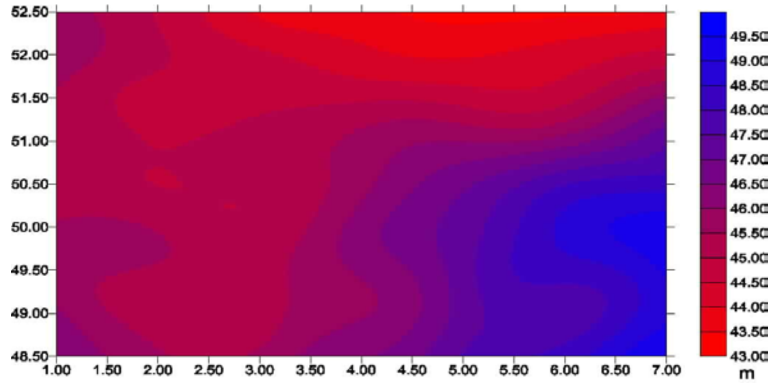

Figure 3b - The EGM96 undulation

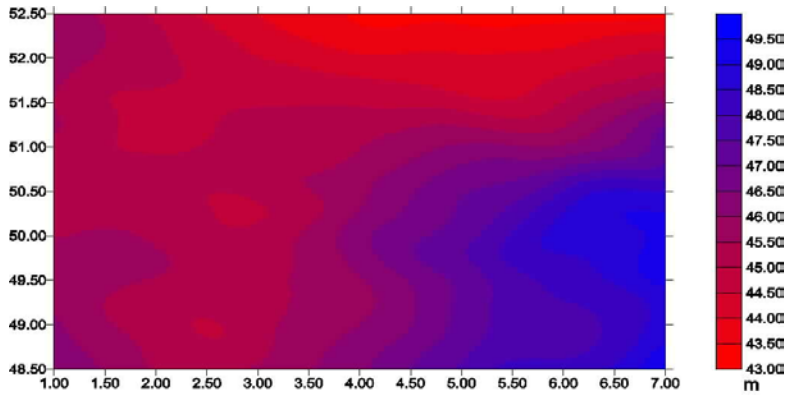

Figure 3c - The GPM98CR undulation

It is quite obvious that OSU91A is close to EGM96 since they have been computed following a similar approach and to the same degree n=360. On the contrary, the Wenzel GPM98CR model is derived following a quite different method and thus differences are expected with respect to OSU91A and EGM96. Furthermore, this model is complete up to degree 720 and it comes from a model which is complete up to degree 1800. Hence, discrepancies in the high frequency content with respect to 360 models are expected too.
Both the geopotential EGM96 model and the GPM98CR model have been used in computing the Belgium quasi-geoid: so, in the end, different estimates will be available to be tested against GPS/leveling data.

2. Estimation procedures and results.

The numerical results related to the estimation procedures are given in the following paragraphs. The classical "remove-restore" (Tscherning, 1994) procedure has been used and the residual quasi-geoid components have been evaluated using the Fast Collocation approach (Bottoni and Barzaghi, 1993) and the FFT technique (Sideris, 1994).

2.1 Quasi-geoid computation and results based on EGM96

The computation of the quasi-geoid named B_EGM96, based on the EGM96 global model, has been carried out on a regular 1' x 1' grid in the area

$$48.5° \le \varphi \le 52.5° \qquad 1° \le \lambda \le 7°$$

With respect to the geopotential model EGM96, the reference DTM for Residual Terrain Correction (RTC) computation has been computed using 25' window size moving average on the detailed DTM. The 25' window size has been tuned on the statistical properties of the residuals with respect to EGM96 model.

RTC has been computed up to 80 km from each computation point both in the gravity and in the quasi-geoid components. Statistics of the "remove" step are listed in tab.1.

Point gravity values have been then gridded on a regular 1' x 1' geographical grid. GEOGRID program of the GRAVSOFT package (Tscherning et al., 1994) was used for such a step: statistics of the residual gridded gravity values Δg_r^G are shown in tab. 1. The empirical covariance of these values and the best fit model, obtained using the COVFIT program (GRAVSOFT), are represented in fig. 4.

As one can see, a satisfactory fit between the empirical values and the model covariance is reached basically up to the first zero. The best fit model, in terms of anomalous potential T(P), has the following general form (Tscherning and Rapp, 1974)

$$COV_{TT}(P,Q) = \sum_{i=2}^{\infty} \sigma_i^2 \left(\frac{R^2}{rr'} \right)^{i+1} P_i(\cos\psi) \qquad (1)$$

where: $\sigma_i^2 = \begin{cases} \text{error degree variances} & i \le 200 \\ \text{degree variances} & \dfrac{A}{(i-1)(i-2)} \left(\dfrac{R_B}{R} \right) \end{cases}$

R = Earth radius R_B = Bjerhammar sphere radius

r, r' = radial distances of points in space P, Q

P_i = Legendre Polynomial of degree i

ψ = spherical distance between P and Q

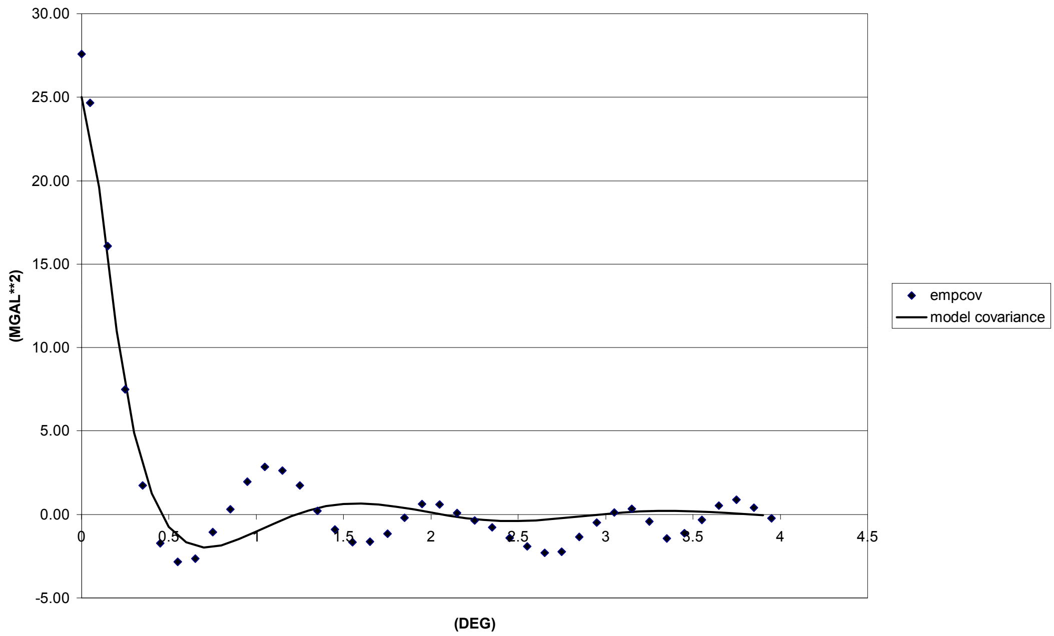

Figure 4 - Empirical and model covariance function of the gridded gravity residuals obtained with the global geopotential model EGM96

	Δg_0 [mGal]	Δg_0 - Δg_M [mGal]	Δg_r [mGal]	Δg_r^G [mGal]
n	43361	43361	43361	87001
E	-1.21	-1.75	-1.78	-1.54
σ	13.69	7.40	6.31	5.27
min	-39.07	-37.25	-37.78	-25.23
max	64.16	36.88	27.87	22.63

Table 1 - Statistics of the "remove" step using the EGM96 geopotential model.

Δg_0: observed gravity values (free air) Δg_M: gravity geopotential model component
A_{rtc} :gravity terrain correction component $\Delta g_r = \Delta g_0 - \Delta g_M - A_{rtc}$ gravity residuals
Δg_r^G: gridded gravity residuals

The Fast Collocation (FC) solution giving ζ_r has been computed on the same 1'x1' grid used for Δg_r^G.
Furthermore, the FFT estimate of ζ_r was also computed to compare the two estimation methods.
The "restore" step was then accomplished: the ζ_{rtc} and the ζ_M component have been added to ζ_r, thus getting the final quasi-geoid estimate B_EGM96. In tab.2 the statistics of the "restore" step are summarized

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	ζ_r (FC) [m]	ζ_r (FFT) [m]	ζ_M (EGM96) [m]	ζ_{RTC} [m]	$\zeta=\zeta_r(FC)+\zeta_M+\zeta_{RTC}$ [m]
n	87001	87001	87001	87001	87001
E	-0.24	-0.37	45.83	0.02	45.61
σ	0.15	0.14	1.35	0.09	1.31
min	-0.58	-0.71	43.28	-0.18	43.16
max	0.08	-0.04	49.33	0.51	49.35

Table 2 - Statistics of the "restore" step using the EGM96 geopotential model

ζ_r : residual quasi-geoid
ζ_M : quasi-geoid geopotential model component
ζ_{rtc} :quasi-geoid terrain correction component
ζ : total quasi-geoid

As one can see, the FC estimate and the FFT solution are practically equivalent but for a bias of 0.13 m. Furthermore, as it is well known, the geopotential model gives nearly the whole quasi-geoid signal, especially in this computation area where no relevant topography and geophysical signals are present.

2.3 Quasi-geoid computation and results based on GPM98CR

In this case, the high resolution geopotential model GPM98CR by Wenzel has been used up to degree 720 to get the B_GPM98CR estimate.

Also in this case, the steps described in the B_EGM96 computation have been performed. The reference DTM in RTC computation was derived by applying a 5' window size moving average on the detailed DTM. As expected, the reference DTM used in this computation differs from the one used in combination with the EGM96 model. Higher frequencies are taken into account when using the GPM98CR geopotential model and so the reference DTM must contain higher frequencies too

Statistics of this "remove" step are given in tab. 3. As done before, residual gravity values have been gridded on a 1'×1' regular geographical grid covering the same area used in the EGM96 based computation (their statistics are listed in tab. 3). The empirical covariance and the best fit model, which belongs to the same kind of function in (1), are shown in fig. 5

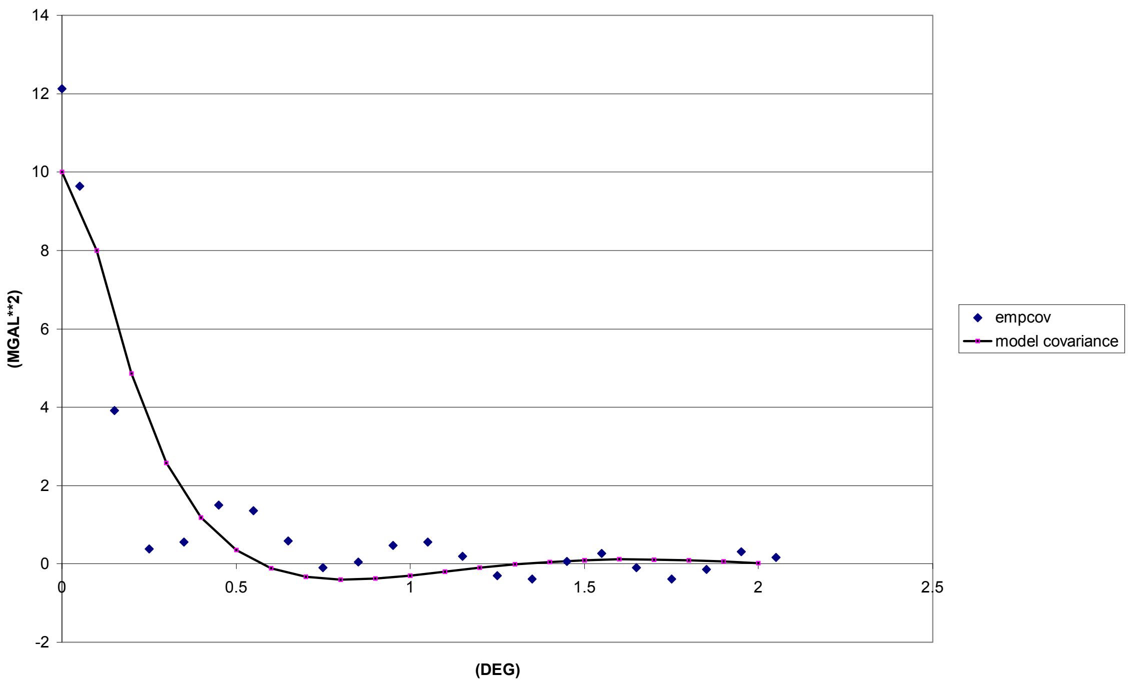

Figure 5 - Empirical and model covariance function of the gridded gravity residuals obtained with the global geopotential model GPM98CR

The empirical covariance is more irregular if compared with the EGM96 empirical covariance but its value in the origin is remarkably smaller that the one obtained for that empirical covariance. This means that the GPM98CR model and the related RTC reduction can give a better representation of the local gravity data than EGM96 (this can be seen also in the statistics of the gravity residuals – compare tab. 1 and tab. 3)

	Δg_0 [mGal]	$\Delta g_0 - \Delta g_M$ [mGal]	Δg_r [mGal]	Δg_r^G [mGal]
N	43361	43361	43361	87001
E	-1.21	-1.62	-1.80	-1.56
σ	13.69	5.00	4.58	3.49
Min	-39.07	-33.93	-32.94	-19.55
Max	64.16	28.08	21.96	12.63

Table 3 - Statistics of the "remove" step using the GPM98CR geopotential model.

Δg_0: observed gravity values (free air)
Δg_M: gravity geopotential model component
A_{rtc} :gravity terrain correction component
$\Delta g_r = \Delta g_0 - \Delta g_M - A_{rtc}$ gravity residuals
Δg_r^G: gridded gravity residuals

As in the previous estimates, Fast Collocation and FFT were applied for computing ζ_r on the 1'x1' regular grid used for Δg_r^G evaluation. The statistics of the "restore" step related to the B_ GPM98CR quasi-geoid are presented in tab. 4.

	ζ_r (FC) [m]	ζ_r (FFT) [m]	ζ_M (GPM98CR) [m]	ζ_{RTC} [m]	$\zeta=\zeta_r(FC)+\zeta_M+\zeta_{RTC}$ [m]
n	87001	87001	87001	87001	87001
E	-0.25	-0.38	45.83	0.02	45.60
σ	0.12	0.11	1.35	0.03	1.31
min	-0.52	-0.64	43.22	-0.80	43.17
max	0.17	-0.03	49.62	0.18	49.43

Table 4 - Statistics of the "restore" step using the GPM98CR geopotential model
ζ_r : residual quasi-geoid
ζ_M : quasi-geoid geopotential model component
ζ_{rtc} :quasi-geoid terrain correction component
ζ : total quasi-geoid

Also for this estimate, the same remarks done for the EGM96 based computation hold.

3. Comparisons with GPS/leveling derived undulations

The two gravimetric quasi-geoid estimates have been compared on 36 points with GPS derived undulations. In these 36 double points, both h (ellipsoidal height) and H (orthometric height) are known so that $N_{GPS/lev}$ = h-H can be computed. Thus, the $N_{GPS/lev}$ values can be compared with the gravimetric estimate to asses its precision. To properly perform the comparison, a datum shift between the gravimetric quasi-geoid estimates and the $N_{GPS/lev}$ must be computed to reduce the data to the same reference system. While $N_{GPS/lev}$ is in the GPS reference system, ζ computed with the “remove-restore” method is in the reference system implied by the global geopotential model.

To this aim, the following formula, which accounts for a translation based datum shift in terms of geoid undulation, has been considered (Heiskanen and Moritz, 1990):

$$\begin{aligned} N_{grav} &= N_{GPS/lev} + \Delta N(\theta,\lambda) = \\ &= N_{GPS/lev} + dx sin\theta \cos\lambda + dy sin\theta sin\lambda + dz\cos\theta \end{aligned} \quad (2)$$

(dx,dy,dz) = translation between GPS and geoid reference systems
$\theta = 90-\varphi$

(we remark that only translation is considered in this relationship between the two reference systems).

In (2), we also assume that $N_{grav} \sim \zeta$, being ζ the quantity which is effectively estimated: this can induce distorsions and perturbations specially in high mountain areas. However, for a first rough relative comparison among the different estimates, we decided to do this assumption, leaving the refinements to further computations.

The quantities (dx,dy,dz) were estimated by least squares; outliers rejection, in the hypothesis of normal distributed residuals and with significance level α =1%, was also performed .Due to that, one GPS point, located in Arlon, was skipped from the solution summarized in the following Table.

	B_EGM96(FC) - $N_{GPS/lev}$	B_EGM96(FFT) - $N_{GPS/lev}$	B_GPM98CR(FC) - $N_{GPS/lev}$	B_GPM98CR(FFT) - $N_{GPS/lev}$
#	35	35	35	35
E	0.00	0.00	0.00	0.00
σ	0.03	0.04	0.03	0.03
min	-0.07	-0.07	-0.07	-0.07
max	0.07	0.07	0.07	0.07

Table 5 - Statistics of the residuals between ζ *and* $N_{GPS/lev}$ *after datum shift estimate*

As it can be seen, the same results have been obtained for the four estimates which are in a very good agreement with the $N_{GPS/lev}$ values. So, it can be concluded that the different estimates in this area are equivalent in representing the undulations coming from GPS and leveling observations.

The plot of the B_EGM96(FC) geoid, named from now on BG03, is shown in the following figure together with the residuals in the 36 double points. In the discussion of the the BG96 estimate, we thought that only the poor coverage of the Ardennes, in the South East of Belgium, was the cause of its low accuracy. One of the conclusions of this work is that it is mainly the point of Arlon, in red on the figure 6, that was responsible of the problems.

Figure 6 –The estimated quasi-geoid BG03 versus $N_{GPS/lev}$

4. Comparison between BG96 and BG03

The fig 7 shows the comparison between BG96 and BG03. In this figure, there is a clear North-South strong gradient at 4.5° of longitude. This anomaly existing only in BG96 is related to the fact that the gravity coverage East of this line was very poor at that time, so that in BG96 the data to the East and to the West of this line were considered as two different data sets. This figure shows also the area improved by BG03. We see clearly large differences up to 4 cm in eastern part between 4.5° and 5.5° longitude and 50° and 50.5° latitude. It is mainly due to the improved gravity coverage. Surprisingly in the South of Belgium the difference between BG96 and BG03 is rather small although the previous gravity coverage was sparse. It is probably due to the fact that in this area most of the signal comes from the DTM. There is of course a very big difference in the south eastern part in 49.5° latitude and 5.8° longitude which is due to the very bad GPS-leveled point in Arlon. It is only in the new of BG03 computation that we could consider this point as an outlier, as this area is now well covered with gravity data. Large changes in the extreme West of the country are probably due to the use of bathymetry data on sea and a better gravity coverage on land in this area.

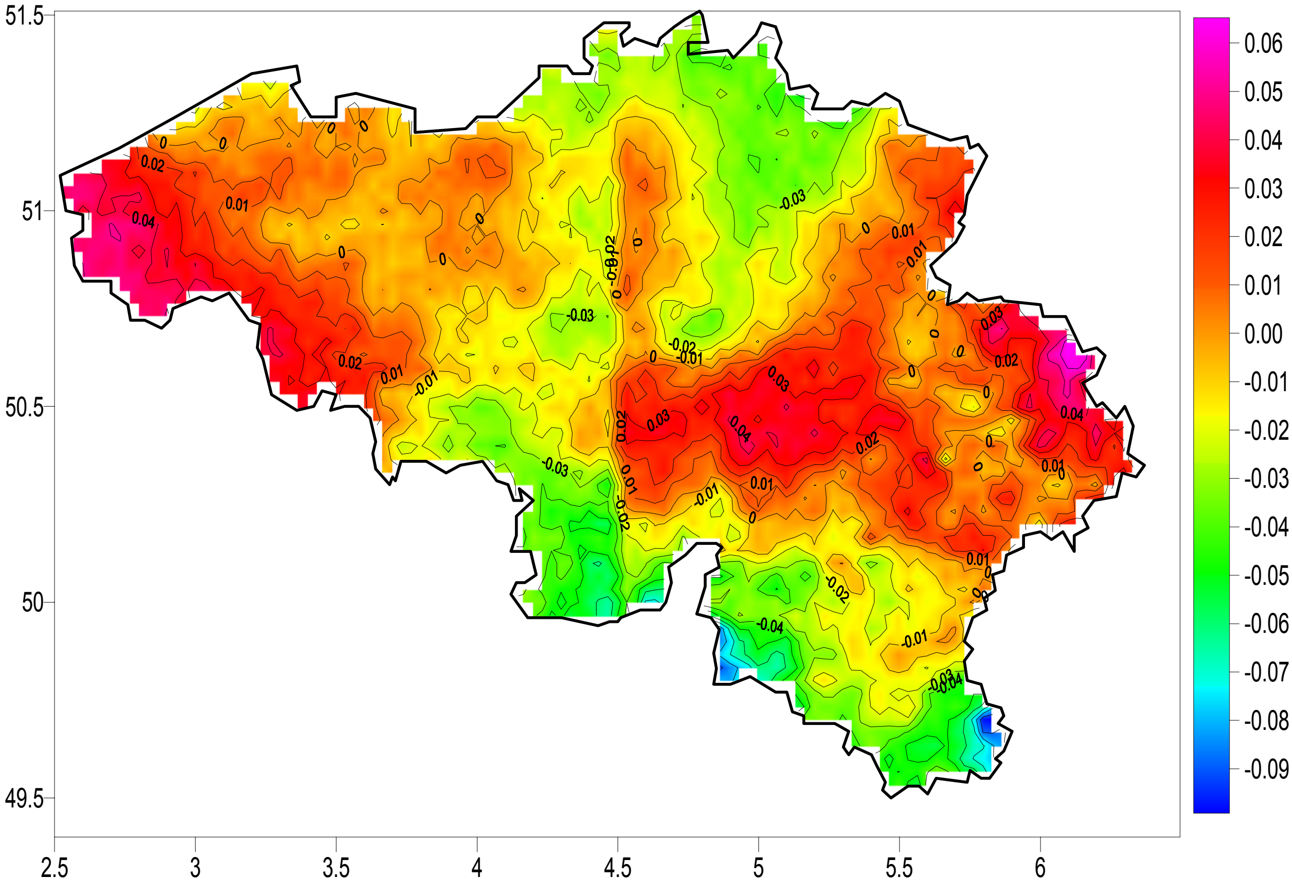

Figure 7 - Comparison between BG96 and BG03 (contour in m)

5. High frequency content of BG03.

Since the gravity coverage is now completed in Belgium, It is interesting to know the areas where the contribution of gravity is more important than the one due to topography. We have thus filtered out from BG03 the low frequency signals. In fig. 8 it can bee seen that, North of 50.5° latitude, where the area is flat, we find back anomalies mainly due to gravity, while South of that line the geoid undulations are mainly due to topography especially in the Eastern part of the country where the highest altitudes are located. We have clearly found back the two main geological units of Belgium, in the North the eroded lower paleozoic and the typical Bouguer anomalies associated to it and in the South the upper paleozoic not yet eroded, where the topography produces the main part of the signal. Let us point out the Flanders anomalies, the EW gravity gradient at the Southern border of the Brabant massif, the Mons basin, the Famennes depression and the main axis of the Ardennes massif.

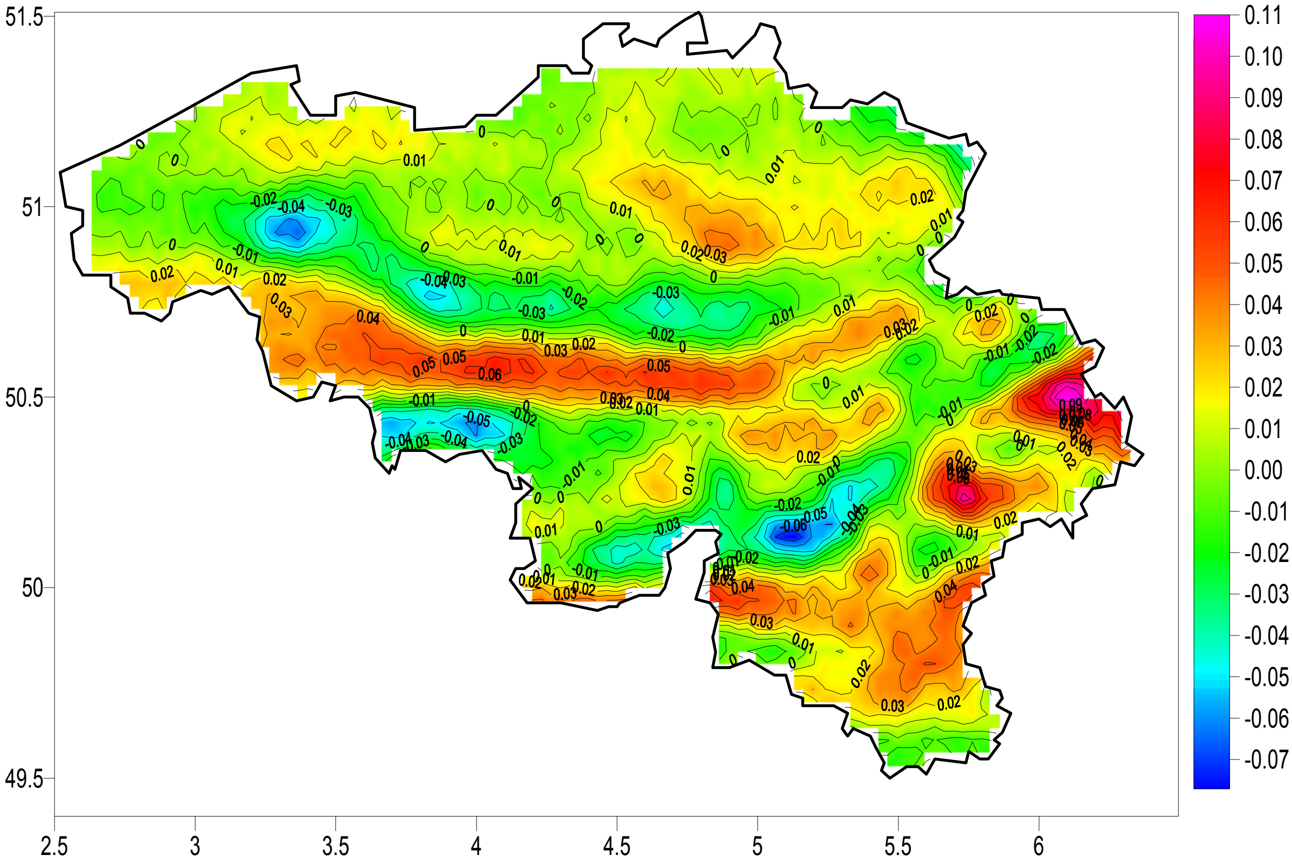

Figure 8 - High frequency content of BG03(contour in m).

6. Conclusions and perspectives

The new quasi-geoid estimate in Belgium is a step forward toward a high precision geoid computation in this area. An improvement has been reached with respect to the previous BG96 solution. This is mainly due to new gravity data, that improved the gravity coverage, more accurate global geopotential models (EGM96 and GPM98CR) and an updated DTM.

Two different techniques, namely FastCollocation and FFT, have been adopted to estimate the residual quasi-geoid component. The obtained results show that, at least for this computation area, the two method are completely equivalent.

The comparisons with $N_{GPS/lev}$ values show that a very good agreement has been reached and prove the obtained refinements in the estimates.

The high frequency content of BG03 is closely connected with the known topographic and geological structures of the Belgian territory.

However, we believe that some efforts must be done to improve the procedure that we adopted to get these solutions.

Particularly, a more detailed DTM should be used to compute a more reliable RTC effect in order to get an homogeneous and isotropic Δg_r field.

Furthermore, the reduction term to transform quasi-geoid into geoid undulations should be also computed to properly compare the gravimetric estimate with $N_{GPS/lev}$ data. Finally, ellipsoidal corrections should be accounted for, although they are more or less constant in the computation window.
It must also be stressed that, in the near future, an integrated quasi-geoid estimate based on gravity and a denser $N_{GPS/lev}$ data set will be computed in the same area.

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